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Modeling and analysis of thermoelastic damping and frequency shift in thin microbeam resonators

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ABSTRACT

This study investigates thermoelastic damping (TED) and frequency shift (FS) in thin microbeam resonators within the framework of the Moore-Gibson-Thompson (MGT) thermoelasticity theory. An explicit formula for thermoelastic damping is derived, and the effects of beam thickness, beam length, isothermal frequency, and thermal relaxation time are analyzed. Numerical results demonstrate that the thermal relaxation parameter plays a significant role in controlling thermoelastic damping and frequency shift at the microscale under different structural and frequency conditions. The findings indicate that the proposed design is suitable for a wide range of damping dissipation applications.

1. Introduction

Many problems can be solved using coupled thermoelasticity theory (Biot, 1956). According to this theory, motion is described by a hyperbolic partial differential equation, and energy conservation is described by a parabolic equation using Fourier's law for heat conduction. Thermal waves propagate at an infinite speed with this type of heat conduction.

In the case of an isotropic body, Lord and Shulman (1967) proposed a generalized thermoelasticity theory with one relaxation time (Lord and Shulman, 1967). Non-Fourier's law replaces Fourier's law with a modified law of heat conduction that includes flux and its time derivative. In this theory, the heat equation associated with the propagation of heat is hyperbolic, thereby eliminating the paradox of infinite speed (Dhaliwal and Sherief, 1980).

A wide range of critical applications, including the processing of mechanical signals, the use of scanning probe microscopes, the detection of ultrasensitive mass, etc., these devices use microresonators. One of the most important parameters of a microresonator is its Q factor, which is also known as the quality factor. This parameter is closely connected to measurement accuracy. Resonators with a high-quality factor dissipate less energy during vibration and are more sensitive. Hence, studying the energy dissipation mechanism is vital to the development and improvement of micro/nanomechanical resonators (Ekinci and Roukes, 2005; Guo et al., 2012).

Zener first investigated thermoelastic damping problems of viscoelastic material and developed his quality factor formula (Lifshitz and Roukes, 2000). According to the classical Fourier thermal conduction theory, Lifshitz and Roukes (2000) developed a formula for thermoelastic damping Q-factor. In their model, there is a peak in thermoelastic damping around the micrometer scale (Guo et al., 2012; Lifshitz and Roukes, 2000). A beam height greater than or smaller than a nanometer will result in a decrease in thermoelastic damping. Experimental results, however, indicate that the Q-factor decreases monotonically with the size of microresonators (Ekinci and Roukes, 2005).

There have been numerous studies on thermoelastic damping, which is calculated using the classical theory of thermoelasticity and Fourier's law for heat conduction (Guo et al., 2012; Lifshitz and Roukes, 2000; Li et al., 2012; Prabhakar and Vengallatore, 2008). According to this theory, Fourier's law offers thermal wave propagation with infinite speeds. To resolve this paradox, several non-classical theories have been developed that permit thermal waves to spread at a finite speed. Adding the heat flux's first-time derivative to Fourier's law of heat conduction results in the second sound theory (Khisaeva and Ostoja-Starzewski, 2006). The generalized thermoelastic theory with one relaxation

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time was applied to the analysis of thermoelastic damping of beam resonators by Sun et al. (2006). According to the Lord-Shulman theory of generalized thermoelasticity theory (L-S) (Sharma and Sharma, 2011), Sharma and Sharma (2011) studied damping in micro-scale circular plate resonators.

In our work, using the Moore-Gibson-Thompson (MGT) model to study the thermoelastic damping (TED) and frequency shift (FS) of a microbeam resonator is a novel study that has not been carried out before, therefore the results will be novel as well. As a result of applying the MGT model, the microbeam resonators become more sensitive and reduce energy dissipation. In another meaning, the MGT introduces high-quality microbeam resonators that reduce energy dissipation during vibration.

2. Problem formulation

2.1. Thermoelasticity equations based on MGT Theory

According to the MGT heat equation, non-Fourier heat conduction occurs as follows (Quintanilla, 2019; Quintanilla, 2020; Kumar and Mukhopadhyay, 2020):

$$\left(\frac{K^*}{K}+\frac{\partial}{\partial t}\right)\left(\frac{\partial^2 T(x,y,t)}{\partial x^2}+\frac{\partial^2 T(x,y,t)}{\partial y^2}\right)=\left(\frac{\partial^2}{\partial t^2}+\tau_q\frac{\partial^3}{\partial t^3}\right)\left[\frac{\rho C_v}{K}T+\frac{T_0(3\lambda+2\mu)\alpha_T}{K}e\right] \quad (1)$$

where, K is the thermal conductivity of the material, $K^*=\frac{(\lambda+2\mu)C_v}{4}$ is the conductivity rate parameter, τ_q is the thermal relaxation time, T is temperature, T_0 is the uniform reference temperature, e is the strain, α_T is the thermal coefficient, C_v is the specific heat, and ρ is the beam density.

Eq. 1 can be written as

$$\left(\frac{K^*}{K}+\frac{\partial}{\partial t}\right)\left(\frac{\partial^2\theta(x,y,t)}{\partial x^2}+\frac{\partial^2\theta(x,y,t)}{\partial y^2}\right)=\frac{\rho C_v}{K}\left(\frac{\partial^2\theta}{\partial t^2}+\tau_q\frac{\partial^3\theta}{\partial t^3}\right)+\frac{T_0(3\lambda+2\mu)\alpha_T}{K}\left(\frac{\partial^2 e}{\partial t^2}+\tau_q\frac{\partial^3 e}{\partial t^3}\right) \quad (2)$$

where, $\theta=T-T_0$ is the temperature change.

2.2. Thermoelastic damping in a microbeam resonator with a rectangular cross-section

The cross-sectional dimension $h\times b$ of an elastic beam is rectangular with a length ℓ, is considered for small flexural vibrations. Take x −axis along the axis of the beam, y −axis along the thickness, and z −axis along the width direction (Fig. 1).

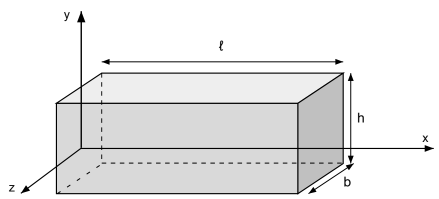


Fig. 1: Rectangular thermoelastic microbeam

The strain takes forms (Guo et al., 2012; Li et al., 2012):

$$e_{xx}=-y\frac{\partial^2 w}{\partial x^2};\ e_{yy}=e_{zz}=\nu y\frac{\partial^2 w}{\partial x^2}+(1+\nu)\alpha_T\theta \quad (3)$$

and

$$e=e_{xx}+e_{yy}+e_{zz}=-y\frac{\partial^2 w}{\partial x^2}+2\nu y\frac{\partial^2 w}{\partial x^2}+2(1+\nu)\alpha_T\theta \quad (4)$$

where, w is the deflection of the beam, ν is Poisson's ratio.

When a beam is free of stress and deformation and kept at a constant temperature T_0, it is in equilibrium. According to the linear Euler-Bernoulli beam theory, the beam deflects due to flexural vibrations in the x-y plane.

So, the beam's equation of motion with thermoelastic coupling is (Guo et al., 2012; Lifshitz and Roukes, 2000; Li et al., 2012):

$$\frac{\partial^2 w}{\partial t^2}+\frac{\partial^2}{\partial x^2}\left(\frac{EI}{\rho A}\frac{\partial^2 w}{\partial x^2}\right)+\frac{E\alpha_T}{\rho A}\frac{\partial^2 I_T}{\partial x^2}=0 \quad (5)$$

$A=h\times b$ is the area of the cross-section, I and I_T are the moment of inertia and thermal moment of the beam, respectively, which are given by (Guo et al., 2012; Lifshitz and Roukes, 2000; Li et al., 2012):

$$I=\iint_A y^2\,dydz \quad (6)$$

and

$$I_T=\iint_A y\theta\,dydz \quad (7)$$

Euler-Bernoulli beams are described by Eq. 2

$$\chi\left(\frac{K^*}{K}+\frac{\partial}{\partial t}\right)\left(\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}\right)=\left(\frac{\partial^2\theta}{\partial t^2}+\tau_q\frac{\partial^3\theta}{\partial t^3}\right)-\frac{\Delta_E(1-2\nu)}{\alpha_T}y\left(\frac{\partial^4 w}{\partial x^2\partial t^2}+\tau_q\frac{\partial^5 w}{\partial x^2\partial t^3}\right) \quad (8)$$

where, $\chi=\frac{K}{\rho C_v}$ is the material thermal diffusivity and $\Delta_E=\frac{T_0E\alpha_T^2}{\rho C_v}$ is Young's modulus for relaxation strength.

The temperature change in a cross-section is much larger along the y −axis than along the x −axis and there are no changes along the z −axis, so we replace ∇^2 with $\frac{\partial^2}{\partial y^2}$. Moreover, $\Delta_E=\frac{T_0E\alpha^2}{\rho C_v}<10^{-6}$ then, Eq. 8 is simplified to

$$\chi\left(\frac{K^*}{K}+\frac{\partial}{\partial t}\right)\left(\frac{\partial^2\theta}{\partial y^2}\right)=\left(\frac{\partial^2\theta}{\partial t^2}+\tau_q\frac{\partial^3\theta}{\partial t^3}\right)-\frac{\Delta_E(1-2\nu)}{\alpha_T}y\left(\frac{\partial^4 w}{\partial x^2\partial t^2}+\tau_q\frac{\partial^5 w}{\partial x^2\partial t^3}\right) \quad (9)$$

To solve Eq. 9, we assumed the beam executed harmonic vibrations with angular frequency ω so that the deflection and the temperature vibration can be expressed as (Guo et al., 2012; Lifshitz and Roukes, 2000; Li et al., 2012):

$$w(x,t)=W(x)e^{i\omega t}\ and\ \theta(x,y,t)=\phi(x,y)e^{i\omega t} \quad (10)$$

by substituting Eq. 10 into Eq. 9 and $a = \frac{K^*}{K}$, we obtain

$$\chi(a + i\omega)\left(\frac{\partial^2\phi}{\partial y^2}\right) = -(\omega^2 + i\tau_q\omega^3)\left(\phi - \frac{\Delta_E(1-2\nu)}{\alpha_T} y \frac{\partial^2 W}{\partial x^2}\right) \quad (11)$$

Eq. 11 can be written as:

$$\frac{\partial^2\phi}{\partial y^2} + \frac{(\omega^2+i\tau_q\omega^3)}{\chi(a+i\omega)}\phi = \frac{(\omega^2+i\tau_q\omega^3)\Delta_E(1-2\nu)y}{\chi\alpha_T(a+i\omega)}\frac{\partial^2 W}{\partial x^2}. \quad (12)$$

Therefore, Eq. 12 has the following solution

$$\phi = A\cos(ky) + B\sin(ky) + \frac{\Delta_E y(1-2\nu)}{\alpha_T}\frac{\partial^2 W}{\partial x^2} \quad (13)$$

where,

$$k = \frac{\omega}{\sqrt{\chi}}\sqrt{a_1 - ia_2} = \frac{\xi}{h}\left(\eta - \frac{i}{\eta}a_2\right)$$
$$\xi = h\frac{\omega}{\sqrt{2\chi}}$$
$$\eta = \sqrt{a_1 + \sqrt{a_1^2 + a_2^2}}$$
$$a_1 = \frac{a+\tau_q\omega^2}{a^2+\omega^2},\ a_2 = \frac{\omega(1-a\tau_q)}{a^2+\omega^2}.$$

We assume that the beam's boundaries are adiabatic (Guo et al., 2012) is

$$\frac{\partial\phi}{\partial y} = 0 \text{ at } y = \pm h/2 \quad (14)$$

The temperatures are distributed across the thickness as follows

$$\phi = \frac{\Delta_E(1-2\nu)}{\alpha_T}\frac{\partial^2 W}{\partial x^2}\left(y - \frac{\sin(ky)}{k\cos(kh/2)}\right) \quad (15)$$

We can now determine the moment of inertia and thermal moment as

$$I = \iint_A y^2\, dydz = \int_0^b \int_{-h/2}^{h/2} y^2 dydz = \frac{bh^3}{12} \quad (16)$$

and

$$I_T = \iint_A y\theta\, dydz = e^{i\omega t}\int_0^b \int_{-h/2}^{h/2} y\phi dydz \quad (17)$$

The thermal moment can be determined using Eq. 15 as follows

$$I_T = e^{i\omega t}\frac{bh^3\Delta_E(1-2\nu)}{12\alpha_T}\left[1 + \frac{24}{h^3k^3}\left(\frac{hk}{2} - \tan\left(\frac{hk}{2}\right)\right)\right]\frac{\partial^2 W}{\partial x^2}. \quad (18)$$

By substituting Eqs. 10 and 18 in Eq. 5, we obtain

$$\omega^2 W = \frac{EI}{\rho A}\left[1 + \Delta_E(1-2\nu)\left[1 + \frac{24}{h^3k^3}\left(\frac{hk}{2} - \tan\left(\frac{hk}{2}\right)\right)\right]\right]\frac{\partial^4 W}{\partial x^4}. \quad (19)$$

By simplifying the last equation, we get the following

$$\omega^2 W = \frac{EI}{\rho A}\left[1 + \Delta_E(1-2\nu)(1 + f(\omega))\right]\frac{\partial^4 W}{\partial x^4}. \quad (20)$$

In this case, the complex function $f(\omega)$ has the following form

$$f(\omega) = f(k(\omega)) = \frac{24}{h^3k^3}\left(\frac{hk}{2} - \tan\left(\frac{hk}{2}\right)\right). \quad (21)$$

The vibration frequency can be driven as follows from Eq. 20

$$\omega = \omega_0\sqrt{1 + \Delta_E(1-2\nu)(1 + f(\omega))}. \quad (22)$$

where, ω_0 denotes that the isothermal frequency values will take the form

$$\omega_0 = q_n^2 h\sqrt{\frac{E}{12\rho}} \quad (23)$$

and

$$q_n L = \{4.73, 7.853, 10.996, \ldots\}, n = 1,2,3, \ldots \quad (24)$$

Eq. 22 can be expanded to the first-order Taylor series as

$$\omega = \omega_0\left[1 + \frac{\Delta_E}{2}(1-2\nu)(1 + f(\omega))\right]. \quad (25)$$

Due to the weak TED, we may replace $f(\omega)$ with $f(\omega_0)$. Then Eq. 25 can be written as

$$\omega = \omega_0\left[1 + \frac{\Delta_E}{2}(1-2\nu)(1 + f(\omega_0))\right]. \quad (26)$$

A TED is calculated as the inverse of a quality factor Q^{-1} (Lifshitz and Roukes, 2000; Li et al., 2012; Sun et al., 2006; Wong et al., 2006) as

$$Q^{-1} = 2\left|\frac{Im(\omega)}{Re(\omega)}\right| \quad (27)$$

The Q^{-1} of the MGT model and the frequency shift Ω can be expressed as:

$$Q^{-1} = 24\Delta_E\frac{a_2}{\xi^2\left(\eta^2 + \frac{a_2^2}{\eta^2}\right)^2} - 24\Delta_E\left[\frac{\left(3\eta a_2 - \frac{a_2^3}{\eta^3}\right)\sin(\eta\xi) - \left(\xi^3 - \frac{3a_2^2}{\eta}\right)\sinh\left(\frac{\xi a_2}{\eta}\right)}{\xi^3\left(\eta^2 + \frac{a_2^2}{\eta^2}\right)^3\left[\cos(\eta\xi) + \cosh\left(\frac{\xi a_2}{\eta}\right)\right]}\right] \quad (28)$$

$$\Omega = \left|\frac{Re(\omega) - \omega_0}{\omega_0}\right| \quad (29)$$

3. Numerical results and discussions

The proposed model MGT is numerically analyzed and plotted using MAPLE software for the following cases (3.1 and 3.2) in this section. To investigate the variation of thermoelastic damping (TED) Q^{-1} and frequency shift (FS) Ω as a result of geometrical and material properties, selected particular values are for beam thickness h, beam length ℓ, and isothermal frequency values ω_0 for different thermal relaxation times $\tau_q = \{0.1, 0.2, 0.3\}$ for a silicon nitride microbeam resonator clamped at two ends.

According to Al-Lehaibi (2022) and Alghamdi (2020), silicon nitride (Si_3N_4) has the following properties:

$T_0 = 293(K), \rho = 3200(kgm^{-3}),$
$K = 43.5(Wm^{-1}k^{-1}), C_v = 630(J.kg^{-1}K^{-1}),$
$\alpha_T = 2.71 \times 10^{-6}(K^{-1}), E = 165(GPa),$
$\nu = 0.22,\ \lambda = 217 \times 10^9 Nm^{-2},\ \mu = 108 \times 10^9 Nm^{-2}$

This section will conduct a comparison study to evaluate the results' reliability, validity, and accuracy.

3.1. Influence of τ_q on TED

In Figs. 2a, 2b, and 2c, the TED is calculated while varying beam thickness h$(10^{-9} \leq h \leq 9 \cdot 10^{-7})$, beam length $\ell(10^{-9} \leq \ell \leq 10^{-6})$, and the isothermal value of frequency $\omega_0(10 \leq \omega_0 \leq 300)$ based on the MGT model with different values of thermal relaxation time $(\tau_q = 0.1,\ 0.2,\ 0.3)$.

According to the analysis, the damping Q^{-1} is negligible if $h,\ \ell,\ \omega_0 \approx 0$, and it reaches a maximum peak value called Q^{-1}_{peak}. Figs. 2a, 2b, and 2c illustrate how the variation Q^{-1} increases rapidly as $h,\ \ell,\ and\ \omega_0$ increase until it reaches a peak and then reverses direction to decrease till the end, then it stabilizes for higher values of $h,\ \ell,$ and ω_0.

This means that the MGT model has less energy dissipation during vibration, resulting in high-quality resonator sensitivity.

In contrast, the variation of thermal relaxation time τ_q affects the behavior of the damping Q^{-1} of a two-sided microbeam clamped at two ends.

3.2. Influence of τ_q on FS

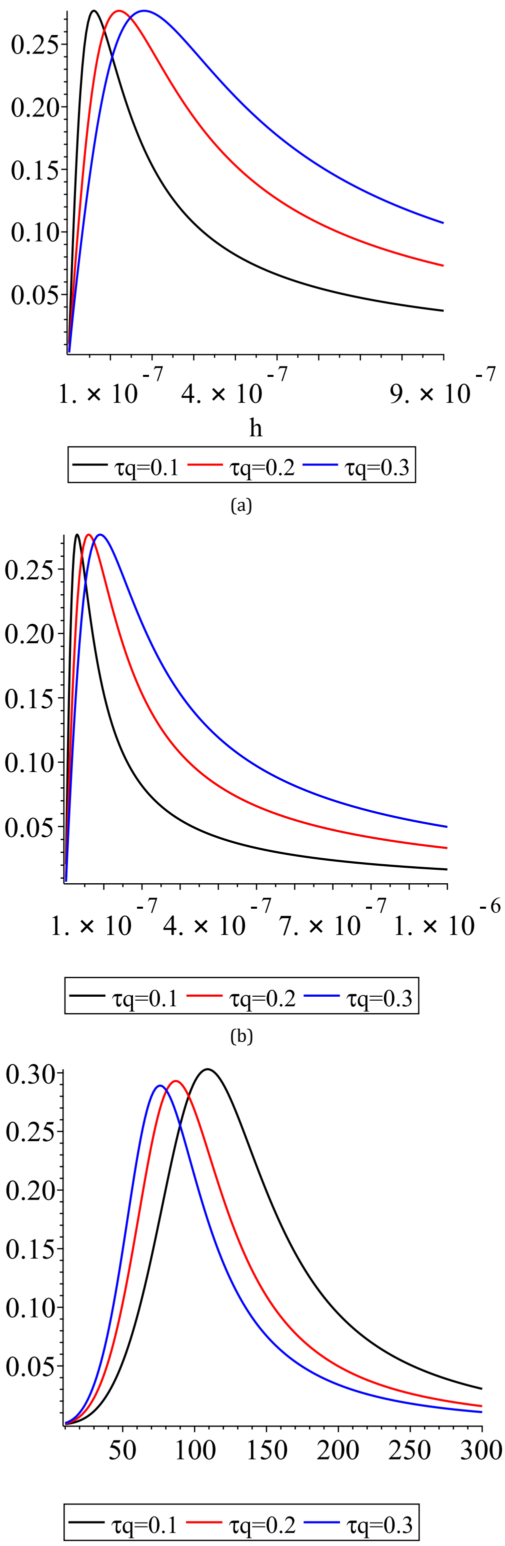
In addition, our study investigated the effect of thermal relaxation time τ_q on the frequency shift FS. According to Figs. 3a, 3b, and 3c, the FS is calculated while varying beam thickness h $(10^{-9} \leq h \leq 9 \cdot 10^{-7})$, beam length $\ell(10^{-9} \leq \ell \leq 10^{-6})$, and isothermal frequency $\omega_0(10 \leq \omega_0 \leq 300)$ based on the MGT model with different values of thermal relaxation time $(\tau_q = 0.1,\ 0.2,\ 0.3)$.

Figs. 3a and 3b show that Ω is negligible as $h,\ \ell \approx 0$. As $h\ and\ \ell$ increase, the variation of Ω increases rapidly, after that Ω is off and stabilizes for higher values of h, and ℓ. In addition, higher values of τ_q lead to lower frequency shifts.

According to Fig. 3c, the variation of Ω start with the maximum stabilizes values for some small values ω_0, then it decreases rapidly with increasing ω_0, until Ω is off and stabilizes again. Also, higher values of τ_q lead to lower frequency shifts.

From Figs. 3a, 3b, and 3c, the results show that the variation of Ω has no peak points in cases of the variation to $h,\ \ell,$ and ω_0. Furthermore, the variation of Ω is off and stabilizes for higher values of $h,\ \ell,$ and ω_0, which represents the phenomenon of energy dissipation.

We conclude from two cases (3.1 and 3.2) that MGT is a new model for high-quality microbeam resonators, where this model decreases energy dissipation. The validity of our previous results was verified by comparing them to those reported in earlier studies (Youssef and Alghamdi, 2015; Alghamdi and Youssef, 2017; Alghamdi, 2016; Zhou et al., 2023; Kharnoob et al., 2024).



(c)

Fig. 2: Variation of TED with respect to thermal relaxation time for (a) thickness, (b) length, and (c) isothermal frequency

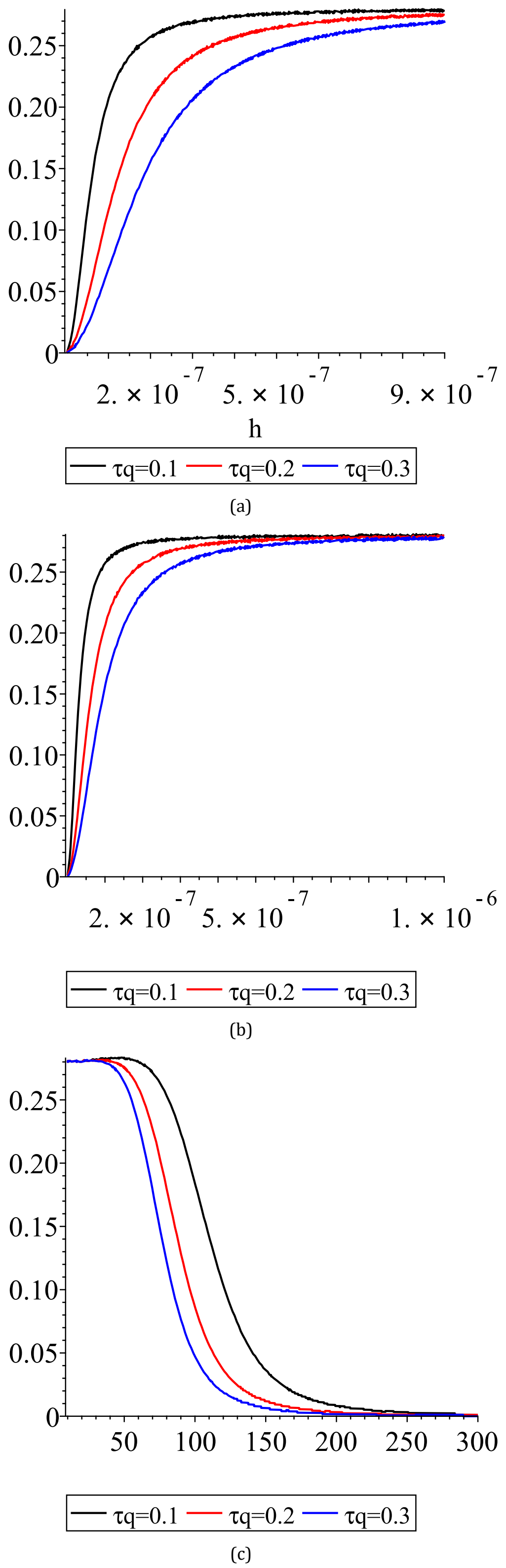


Fig 3: Variation of FS with respect to thermal relaxation time for (a) thickness, (b) length, and (c) isothermal frequency

4. Conclusion

In our work, a thermoelastic damping TED is derived analytically for small vibrations of a thin elastic microbeam resonator with a rectangular cross-section, and we conclude that:

1. Relaxation time parameters play a significant role in TED in the presence of thickness variations h, length variations ℓ, and isothermal frequency variations ω_0.
2. Relaxation time parameters play a significant role in FS in the presence of thick variations h, length variations ℓ, and isothermal frequency variations ω_0.
3. In microbeam resonators, the Moore-Gibson-Thompson Thermoelasticity model reduces the amount of energy dissipated during flexural vibration and improves their sensitivity.
4. As a result of the Moore-Gibson-Thompson Thermoelasticity theory, high-quality microbeam resonators are introduced where energy dissipation is decreased during vibration.

List of symbols

K	Thermal conductivity
K_r	Conductivity rate parameter
q	Heat flux
τ	Thermal relaxation time
T	Temperature
T_0	Reference temperature
α	Coefficient of thermal expansion
C	Specific heat
ρ	Material density
θ	Temperature change
h	Beam thickness
L	Beam length
A	Cross-sectional area
e	Strain
w	Beam deflection
ν	Poisson's ratio
I	Moment of inertia
I_T	Thermal moment
χ	Thermal diffusivity
E	Young's modulus
ω_0	Isothermal frequency

Compliance with ethical standards

Conflict of interest

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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