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Integration of the TPACK approach and augmented reality technology in a hybrid learning model to improve the understanding of geometry concepts

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ABSTRACT

This study develops and evaluates the effectiveness of a hybrid learning model based on Technological Pedagogical Content Knowledge (TPACK) and Augmented Reality (AR) to improve junior high school students' understanding of geometry concepts. The study addresses low student achievement in geometry, particularly in understanding shapes, their properties, and the calculations of three-dimensional figures. A Design-Based Research (DBR) approach was applied through five stages: problem identification, solution design, development and validation, limited implementation, and evaluation. The participants were 80 eighth-grade students divided into an experimental group and a control group, each consisting of 40 students. Expert validation showed that the learning materials and AR media were highly valid, with an average score of 87.3%. The model was implemented over 10 learning sessions using a combination of face-to-face and online instruction supported by AR media and a learning management system. The results showed that the experimental group achieved significantly higher conceptual understanding than the control group, with an average N-Gain score of 0.78 (high category) compared to 0.41 (medium category). Statistical analysis using a t-test confirmed a significant difference between the two groups. Student questionnaire results also indicated high satisfaction and learning engagement. These findings demonstrate that AR-based hybrid learning effectively enhances students' spatial visualization and understanding of geometry concepts and supports the integration of digital technology in 21st-century education.

1. Introduction

Geometry is one of the main pillars in the primary and secondary school mathematics curriculum (Narasimhamurthy and Praveen, 2024; Yao, 2020). This subject not only develops calculation skills but also hones spatial thinking, visualization, and logical reasoning abilities (Akremi et al., 2024; Tripathi and Lee, 2025; Wampler and Plecnik, 2025). However, geometry is often considered abstract material and difficult for students to understand because it requires managing mental representations of three-dimensional objects (De La Rosa and Ruiz, 2025; Elzohbi and Zhao, 2024; Heinrich et al., 2025). Understanding geometry concepts is one of the important competencies in learning mathematics, which is still a big challenge for junior high school students (Caswell et al., 2025; Chung and Abbott, 2021; Libertus et al., 2024). The characteristics of abstract and visual geometry materials require a learning approach that can facilitate spatial and conceptual thinking processes concretely (Kim et al., 2025; Praveen and Narasimhamurthy, 2025; Sarfaraz et al., 2024; Zhu et al., 2025). Many students have difficulty understanding concepts such as points, lines, angles,

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planes, as well as the properties of flat and spatial shapes (Alharbi, 2025). Conventional visual representations, such as static images in books or on whiteboards, are often insufficient to help students form a concrete picture of geometric structures (Xu et al., 2021). The gap between mathematical abstraction and students' visual imagination becomes a major obstacle in learning (Huang et al., 2023).

As educational technology develops, Augmented Reality (AR) emerges as an innovative solution. AR enables the integration of 3D digital objects into the real world through devices such as smartphones or tablets, so that students can manipulate geometric shapes in real-time and interactively. AR brings the benefits of more concrete and intuitive visualization, which is very relevant for geometry materials (Pujiastuti and Haryadi, 2024).

The results of the study explained that the use of AR in geometry learning successfully improved students' spatial visualization skills significantly (Pujiastuti and Haryadi, 2023a). Students who were taught with AR were able to better understand the concept of volume and surface area of spatial shapes compared to using conventional methods, such as two-dimensional diagrams. Other research also shows that AR not only increases learning motivation but also strengthens conceptual understanding, retention, and student engagement. These positive effects are strongest in abstract materials that require three-dimensional visualization, such as geometry (Bujak et al., 2013).

It is noted that several studies have developed AR media for geometry with a systematic approach. The results show that the use of AR GeoGebra shows the effectiveness of AR in learning basic geometry (Del Cerro Velázquez and Morales Méndez, 2021). Furthermore, the results showed that interactive AR media developed through the ADDIE stages significantly improved learning outcomes and student interest. Another study revealed that AR can improve students' understanding of the concept of surface area and volume of prisms. Real-time interaction in three-dimensional form provides a more concrete learning experience and improves the quality of students' conceptual understanding (Cabero-Almenara et al., 2019).

In the context of geometry learning, teachers can design activities such as 3D building exploration, AR object manipulation, and visual-based concept discussions. This process allows students to directly experience concepts such as the relationship between sides, angles, nets, area, and volume in a real and interactive constellation. However, the effectiveness of AR depends on the quality of instructional and pedagogical design. AR without a clear pedagogical direction has the potential to become mere visual entertainment. Therefore, technology integration must be based on the Technological Pedagogical Content Knowledge (TPACK) framework, where teachers have in-depth knowledge of material content, teaching strategies, and synergistic use of technology.

The integration of TPACK and Augmented Reality (AR) in learning is an innovative approach that strategically positions technology in the teaching and learning process, without neglecting pedagogical aspects and material content. TPACK (Technological Pedagogical Content Knowledge) is a conceptual framework that emphasizes the importance of a balance between three main components: technological knowledge, pedagogical knowledge, and understanding of content (Batool et al., 2025; Heath and Moore, 2024; Mohammadpour and Maroofi, 2025). In the context of geometry learning at the junior high school level, the use of AR allows teachers to display three-dimensional visual representations of geometric shapes directly through digital devices, such as smartphones or tablets.

The use of AR in a TPACK-based learning model allows students to better understand the shape, size, and structure of geometric shapes concretely and interactively (Iftene and Trandabăţ, 2018). This supports the content (Content Knowledge) taught using a more participatory pedagogical approach (Pedagogical Knowledge), such as group discussions, independent exploration, and guided discovery. Technology (Technological Knowledge) here acts as a bridge between conceptual abstraction and visual reality that students can manipulate.

TPACK emphasizes the integration between Content Knowledge (CK), Pedagogical Knowledge (PK), and Technological Knowledge (TK), and their overlap in the form of PCK, TCK, and TPK (Schubatzky et al., 2025; Shambare and Simuja, 2024). When teachers can implement AR based on a comprehensive understanding of these aspects, learning becomes more meaningful and effective. Although AR displays great potential, some challenges often arise, such as infrastructure readiness, devices, as well and students' cognitive load if the AR design is not carefully designed. These challenges require a mature pedagogical approach, among others, through the TPACK framework and hybrid learning models.

The integration of TPACK and Augmented Reality (AR) becomes even more effective when applied in a hybrid learning model, which synergistically combines face-to-face and online learning. In this context, TPACK provides a conceptual framework for teachers to design lessons that effectively integrate technology (such as AR) in delivering geometry content, while also selecting pedagogical strategies appropriate to student characteristics and the learning environment (Pujiastuti and Haryadi, 2023b).

In face-to-face learning, teachers can facilitate exploration of geometric shapes using AR media to help students visualize three-dimensional shapes in a realistic and manipulative way. Meanwhile, in online learning, students can still access AR media independently via smartphones or other devices through a Learning Management System (LMS) such as Google Classroom, complete with digital worksheets, learning videos, and interactive quizzes.

This combination reflects the practical application of Technological Knowledge (TK) in supporting Content Knowledge (CK) and Pedagogical Knowledge (PK).

The hybrid learning model allows for learning flexibility and broader access to learning resources. By integrating TPACK and AR, this model not only increases student engagement in the learning process but also fosters deeper conceptual understanding and accommodates diverse learning styles. Learning becomes not only technologically adaptive but also pedagogically relevant and content-based.

This study offers a new contribution to the world of mathematics education by developing and empirically testing a TPACK-based hybrid learning model supported by Augmented Reality (AR) media to improve junior high school students' understanding of geometry concepts. Although AR has been widely used in various learning contexts, its full integration in a TPACK-based pedagogical model implemented in a hybrid learning scheme is still very rare, especially on the topic of geometry at the junior high school level in Indonesia.

The novelty of this study lies in three aspects: (1) synthesizing the TPACK framework, AR technology, and the hybrid learning model into one comprehensive instructional intervention; (2) developing contextual AR media specifically designed to visualize spatial geometry concepts in three dimensions, enabling direct object manipulation in a pedagogically guided learning environment; and (3) applying a hybrid learning model that combines synchronous (face-to-face) and asynchronous (independent online) sessions to enhance flexibility, engagement, and depth of learning.

Overall, this research not only brings innovation in terms of instructional design and media development but also offers a learning model that can be replicated and further developed by teachers and other researchers in various subject contexts and educational levels. In addition, this research is also a breakthrough because it was conducted in a local context that is relevant and has not been widely researched, namely junior high school students in Banten Province, Indonesia. This research has the potential to make a practical contribution to the development of technology-based learning strategies in areas where academic achievement still needs to be improved.

This study aims to develop and test the effectiveness of a hybrid learning model assisted by Augmented Reality media, designed based on the TPACK framework, in improving junior high school students' understanding of geometry concepts. The main objective of this research is to produce an innovative learning model that is not only theoretically and practically feasible but also able to significantly improve student learning outcomes. Specifically, this research seeks to (1) design a TPACK-based hybrid learning model integrated with AR media, (2) test the feasibility of the model in terms of material, media, and implementation, (3) evaluate the effectiveness of the model on improving students' understanding of geometry concepts, and (4) identify student responses to the use of AR media in learning. With this approach, the research is expected to make a theoretical contribution in the development of 21st-century learning models, as well as a practical reference for teachers in improving the quality of mathematics learning. In addition to improving understanding of geometry concepts, AR-based hybrid learning also supports learning differentiation. Students can learn at their own pace and style and repeat the use of interactive media when facing difficulties, thus strengthening long-term understanding.

2. Methods

This research uses the Design-Based Research (DBR) approach, which is a systematic approach designed to produce learning innovations through iterative cycles of design, implementation, and reflection (Peschl et al., 2023). This approach is very relevant in developing hybrid learning models based on TPACK and Augmented Reality (AR) technology, because DBR not only evaluates effectiveness, but also develops solutions based on real practices in the field (Yang and Lee, 2025).

The research design was structured to systematically develop, implement, and evaluate a hybrid learning model based on the Technological Pedagogical Content Knowledge (TPACK) framework and supported by Augmented Reality (AR). Each stage informed the next, allowing refinement of the instructional model based on empirical evidence and reflection.

The first stage focused on problem identification and analysis. At this stage, the researchers examined the existing conditions of geometry learning by conducting classroom observations, interviewing mathematics teachers, and reviewing students' geometry achievement data from the previous three academic years. The purpose of this stage was to identify persistent learning difficulties, particularly those related to spatial visualization and conceptual understanding of three-dimensional geometry.

The second stage involved the design and development of the learning model. Based on the findings from the initial analysis, a hybrid learning model integrating face-to-face and online learning was designed. Augmented Reality media were developed to support the visualization of spatial geometry concepts, and learning tools such as lesson plans, digital worksheets, instructional videos, and learning management system materials were prepared. All instructional components were designed in alignment with the TPACK framework to ensure coherence between content knowledge, pedagogical strategies, and technological support.

The third stage was the implementation of the developed learning model in a real classroom context. The hybrid learning model assisted by AR was applied to an experimental group of eighth-grade students, while a control group received conventional instruction. During this stage, the learning process was observed, and data related to student engagement, learning activities, and classroom interactions were collected to capture how the model functioned in practice.

The final stage consisted of reflection and evaluation. At this stage, the effectiveness of the learning model was evaluated through analysis of pretest and posttest results, measurement of learning gains, and statistical testing to determine the significance of observed differences. In addition, student responses and expert feedback were analyzed to reflect on the strengths and limitations of the model. The outcomes of this stage were used to assess the impact of the intervention and to inform possible refinements of the instructional design.

At the problem identification and analysis stage, the research focused on understanding the existing conditions of geometry learning. Classroom observations were conducted to examine conventional instructional practices and students' learning behaviors. In addition, interviews with teachers and students were carried out to reveal common difficulties in understanding geometry concepts. To strengthen these findings, document analysis of student learning outcomes was performed, including daily test scores and semester examination results from the previous three academic years.

At the design and development stage, the learning model was systematically constructed based on the results of the problem analysis. A hybrid learning model combining face-to-face and online learning activities was developed to support flexible and effective instruction. In parallel, three-dimensional Augmented Reality geometry media were created using Unity and Vuforia-based applications to visualize spatial geometry concepts in an interactive manner. In addition, TPACK-based instructional tools were developed to support the learning process, including digital student worksheets delivered through Google Forms, interactive instructional videos, a structured geometry module, and learning materials organized within a learning management system using Google Classroom. To ensure the quality and feasibility of the developed model and learning tools, an expert validation process was conducted involving three specialists, consisting of a geometry content expert, a learning media expert, and a pedagogy and technology expert.

At the implementation stage, the research was conducted in a purposively selected public junior high school in Banten Province, Indonesia. The participants consisted of two classes of eighth-grade students, with each class comprising 40 students. One class served as the experimental group and received instruction using the TPACK-based hybrid learning model supported by Augmented Reality, while the other class functioned as the control group and received conventional instruction. In addition to student participants, the implementation process also involved mathematics teachers and three experts who contributed to the validation of learning materials, media, and pedagogical design. Each class consisted of an equal gender distribution, with 20 male and 20 female students, resulting in a total of 80 student participants. This proportional distribution was applied to minimize potential gender bias in the comparison of learning outcomes and student responses.

During implementation, learning outcomes and student perceptions were analyzed not only in terms of score improvement but also by considering gender distribution to identify possible differences in responses to the learning model. The instructional intervention was carried out over ten learning sessions that combined synchronous face-to-face instruction and asynchronous online activities. The learning process began with a pretest to measure students' initial understanding of geometry concepts, followed by an orientation session introducing the Augmented Reality media and its technical use. Subsequent face-to-face sessions focused on presenting and exploring spatial geometry topics such as cubes, cuboids, prisms, and pyramids using AR visualization to support conceptual understanding. Asynchronous sessions allowed students to explore AR models independently through personal devices, complete digital worksheets via the learning management system, and engage in reflective and project-based activities. Group discussions, presentations, interactive quizzes, and contextual problem-solving activities were integrated throughout the sessions to reinforce learning. The instructional sequence concluded with a posttest to assess learning improvement and a questionnaire to capture students' perceptions of the hybrid learning model.

Each learning session was conducted through a combination of online platforms, including the learning management system and instructional videos, and face-to-face activities supported by AR media accessed through smartphones or tablet devices. Data collection instruments used during implementation included an engagement observation sheet to record student participation during learning activities and a student response questionnaire based on a Likert scale to measure engagement, motivation, and perceptions toward the learning model.

At the reflection and evaluation stage, the collected data were systematically analyzed to assess the effectiveness of the developed learning model. Reflection data were examined to understand students' learning awareness and experiences throughout the intervention. Learning effectiveness was evaluated through analysis of pretest and posttest results based on descriptive test items measuring conceptual understanding. Statistical analyses were conducted, including normality and homogeneity tests, followed by an independent t-test to determine the significance of differences between the experimental and control groups. In addition, learning improvement was measured using N-Gain analysis to evaluate the effectiveness of the hybrid learning model assisted by Augmented Reality in enhancing students' understanding of geometry concepts.

3. Results and discussion

This research develops and tests the effectiveness of a TPACK-based hybrid learning model supported by Augmented Reality to improve junior high school students' understanding of geometry concepts. The study employs a Design-Based Research approach that is carried out through four sequential and interconnected stages, beginning with problem identification and analysis, followed by the design and development of the learning model, continuing with implementation in a real classroom context, and concluding with reflection and evaluation to assess effectiveness and inform refinement of the instructional design.

At the problem identification and analysis stage, the findings were derived from three main sources: direct classroom observation, in-depth interviews with mathematics teachers, and document analysis of students' geometry test scores over the previous three academic years. Together, these data sources provided a comprehensive picture of the main difficulties faced by students in understanding spatial geometry concepts.

Classroom observations were conducted during three geometry learning sessions with a focus on student engagement and conceptual understanding of three-dimensional shapes. The observations revealed that a majority of students experienced difficulties in understanding the relationships between geometric elements such as faces, edges, and vertices. Many students also struggled to visualize three-dimensional shapes when they were presented only through two-dimensional representations in textbooks. In addition, a large proportion of students showed low participation when asked to solve problems related to surface area and volume, and only a small number of students were able to confidently explain the relationship between geometric concepts and the context of story-based problems. Qualitative findings from teacher interviews further reinforced the results of classroom observations. Teachers reported that students tended to memorize formulas without developing a deep understanding of basic geometry concepts. When faced with contextual or story problems, students often failed to translate verbal information into appropriate geometric models. Teachers also noted that many students were unable to distinguish between different types of spatial figures, such as prisms and pyramids, despite being provided with visual illustrations. According to the teachers, the learning media used in class had largely been static and lacked interactive elements, resulting in low levels of visual and conceptual engagement among students.

Document analysis of students' geometry examination results over the previous three academic years further confirmed these findings. As summarized in Table 1, student performance in geometry remained consistently low across the observed period, with a substantial proportion of students failing to achieve the minimum completion criteria, particularly on topics related to spatial figures. This trend indicates that difficulties in understanding geometry concepts are persistent and systemic, aligning with the results of classroom observations and teacher interviews.

On average over the previous three academic years, approximately 71% of students did not achieve the minimum completion score of 75 in geometry, particularly on topics related to spatial shapes. These findings confirm that a large proportion of students, estimated at around 70%, experience fundamental difficulties in understanding geometry concepts from structural, numerical, and applicative perspectives. Evidence from classroom observations, teacher interviews, and analysis of examination results consistently indicates that these learning difficulties are widespread and persistent.

Based on these identified problems, the study proceeded to the design and development stage, during which a new learning model was constructed to address the observed deficiencies in geometry learning. The framework of the newly developed learning model, which integrates the TPACK approach with a hybrid learning strategy supported by Augmented Reality, is presented in Table 2.

Table 1: Geometry exam scores

School year	Number of students	Score ≥ 75	Score < 75	Percentage of students not completed
2021/2022	160	51	109	68.1%
2022/2023	158	45	113	71.5%
2023/2024	161	43	118	73.3%

As part of the design and development process, short instructional videos were developed as core learning resources that students could access independently. Each video focused on a specific topic in spatial geometry and presented three-dimensional visualizations through Augmented Reality technology. The videos were designed with a short duration of approximately three to five minutes to match the attention span of junior high school students. Geometry objects were displayed in rotational three-dimensional form, accompanied by concise explanations supported by text and animation. Reflection pauses were embedded within the videos to encourage active thinking and conceptual understanding, and all video materials were integrated into Google Forms and Google Classroom to support both guided and independent learning.

In addition to instructional videos, Google Classroom was used as a learning management system to integrate all learning components within a single platform.

This system facilitated the distribution of learning materials, collection of assignments, organization of discussion forums, and implementation of learning assessments. The structure of the interactive learner worksheets delivered through Google Forms is summarized in Table 3, while the overall organization of the learning management system is presented in Table 4. Through the use of the learning management system, teachers were able to organize hybrid learning activities effectively and monitor student participation in both online and face-to-face modes. In this integrated learning environment, digital worksheets functioned as tools for practice and evaluation, instructional videos served as interactive visual learning resources, and the learning management system acted as the central coordinator of all learning activities.

Table 2: Learning model syntax of the TPACK-based hybrid learning model

Phase	Learning activity	Description	Main media/technology
Orientation and motivation	Initial engagement	The teacher introduces learning objectives and motivates students using short videos or three-dimensional geometry animations to activate prior knowledge.	Instructional video, projector
AR exploration	Concept visualization	Students explore three-dimensional geometry objects using Augmented Reality to observe structures, elements, and spatial relationships interactively.	AR application (Unity–Vuforia), smartphones/tablets
Concept discussion and elaboration	Concept construction	Students discuss observations from AR exploration in groups, clarify concepts, and reinforce relationships among geometric elements with teacher guidance.	AR media, classroom discussion
Contextual application	Problem solving	Students solve contextual geometry problems related to real-life situations using digital worksheets to apply learned concepts.	Digital LKPD (Google Forms), LMS
Reflection and feedback	Learning evaluation	Students reflect on their understanding and learning process, while teachers provide formative feedback to strengthen conceptual mastery.	LMS (Google Classroom), online feedback tools

Table 3: Interactive learner worksheet based on Google Forms

Section	Description	Learning objective
Apperception	AR cube illustration and triggering a question	Activate students' prior knowledge
Exploration	Visual and animation-based questions	Recognize the elements of building a space
Elaboration	Contextual problem solving	Applying concepts in a real-life context
Reflection	Comprehension rating and open-ended questions	Developing awareness of self-learning

Table 4: Structure of LMS

Component	Function
Stream	Deliver hybrid learning information and announcements
Classwork	Divided by the geometry topic, containing assignments and materials
Materials	Stores modules, videos, AR links, and LKPDs
Assignments	Collection of independent and group assignments
Questions	Discussion forum and end-of-learning reflection

As part of the design and development process, the learning model, instructional media, and assessment tools were evaluated through an expert validation process involving three specialists with expertise in educational media, mathematics content, and pedagogy and technology related to the TPACK framework. This validation was conducted to assess the feasibility, coherence, and quality of the developed learning components.

The results of the expert validation are presented in Table 5. The findings indicate that the suitability of the TPACK-based learning syntax was rated very highly by all experts, with an average score of 87.7%, reflecting strong integration between technological, pedagogical, and content elements. The Augmented Reality media were also judged to be feasible and easy to operate for junior high school students, achieving an average score of 87.7%, which confirms their appropriateness for supporting visual geometry learning. In addition, the learning tools, including lesson plans, digital worksheets, and instructional modules, obtained an average score of 87.3%, indicating that they met key criteria related to content accuracy, presentation quality, language clarity, and usability within a hybrid learning context. Although the overall evaluation was highly positive, experts provided minor recommendations, particularly regarding the need to shorten video duration and to provide clearer instructions for the use of AR media. Overall, the validation results demonstrate that the TPACK-based hybrid learning model supported by Augmented Reality is highly valid and ready for classroom implementation, while also being well aligned with 21st-century learning principles emphasizing flexibility, collaboration, and interactivity.

At the implementation stage, student engagement was systematically observed over ten learning sessions, focusing on cognitive, affective, and psychomotor aspects of participation. The results of these observations for the experimental group are summarized in Table 6.

Based on the results presented in Table 6, most students demonstrated a high level of interest when learning with Augmented Reality. During the learning sessions, students actively asked questions, interacted with the AR media, and explored the visualized spatial shapes, indicating strong engagement in the learning process.

To further examine students' perceptions of the learning model, a response questionnaire consisting of ten statements was administered. The questionnaire measured students' perceptions of the learning media, instructional methods, and their impact on conceptual understanding. The results of the student response questionnaire are summarized in Table 7.

The findings indicate that students' understanding of spatial geometry concepts through the use of AR received very high ratings, with average scores ranging from 4.7 to 4.8, demonstrating strong alignment with the research objectives. Students' learning motivation and confidence also increased noticeably, with average scores between 4.3 and 4.6, reinforcing the effectiveness of the interactive hybrid learning approach. Overall, the average questionnaire score was approximately 4.5, falling within the "strongly agree" category and reflecting students' positive responses to the developed learning model. The effectiveness of TPACK integration was further evident from high scores related to the use of Google Classroom, digital worksheets, and self-directed learning activities.

Table 5: Results of validation of the main aspects of the learning model

Aspects assessed	Expert 1 (%)	Expert 2 (%)	Expert 3 (%)	Average (%)
Appropriateness of TPACK syntax	86	89	88	87.7
Feasibility of AR media	88	86	89	87.7
Quality of learning tools (lesson plans, LKPD, modules)	87	86	89	87.3
Ease of media use	85	88	89	87.3
Appropriateness of the hybrid learning format	88	86	88	87.3

Table 6: Percentage of student engagement (experiment class)

Aspect	Indicator	Average (%)	Category
Cognitive	Focused, actively asking questions, and answering	87.50%	Very active
Affective	Enthusiasm, cooperation, and expression	85.20%	Very active
Psychomotor	AR object manipulation, independent practice	89.10%	Very active

Following the implementation phase, students were asked to complete a reflection activity to evaluate their learning experiences. This reflection was administered after the posttest using a five-point scale ranging from strongly disagree to strongly agree. Five reflection statements were designed to assess students' learning awareness, metacognitive engagement, and perceptions of their learning process. The results of the student reflection analysis are presented in Table 8.

Table 7: Student response questionnaire

No.	Statement	Average score (Scale 1-5)	Category
1	AR media helps me understand the concept of building space more realistically.	4.7	Strongly agree
2	I find it easier to imagine geometric shapes in 3D	4.6	Strongly agree
3	Learning with AR is more interesting than regular learning.	4.8	Strongly agree
4	Video and digital LKPD help me learn independently at home	4.5	Strongly agree
5	Google Classroom makes it easy for me to access materials and assignments.	4.4	Agree
6	I feel more confident in doing geometry problems	4.3	Agree
7	I am more motivated to learn geometry with a hybrid model like this.	4.6	Strongly agree
8	The combination of online and offline learning helps me focus more.	4.5	Strongly agree
9	AR media makes it easier for me to understand geometry story problems.	4.4	Agree
10	I hope this kind of learning is also used in other lessons	4.7	Strongly agree

Table 8: Student reflection results

No.	Reflection Statement	Average score	Category
1	I understand better because I can see geometry objects in real life.	4.7	Strongly agree
2	I realize that I must learn actively, not just watch.	4.4	Agree
3	I know which part of the material I haven't mastered yet.	4.3	Agree
4	I can relate the material to real problems	4.5	Strongly agree
5	I feel learning with this media makes me think more deeply	4.6	Strongly agree

The results presented in Table 8 indicate that students were able to evaluate their own learning independently, recognize their strengths and weaknesses, and perceive the benefits of visual and interactive learning approaches. In addition to the structured reflection items, students were also given the opportunity to express their opinions openly after completing the entire learning process.

Students' written reflections further illustrate the impact of the learning model. Several students reported that previously they had difficulty understanding geometry because they were unable to imagine spatial shapes, but that seeing the shapes directly through Augmented Reality helped them understand the importance of geometric elements such as faces and edges. Other students expressed increased enthusiasm for learning, describing the experience as engaging and similar to playing a game while still learning. Some students acknowledged specific conceptual difficulties, such as understanding the volume of pyramids, and explained that the availability of instructional videos allowed them to revisit the material and seek clarification during class. Students also noted that the hybrid learning format enabled them to learn more gradually and reduced anxiety related to examinations. Overall, these reflections indicate that the use of AR media supported spatial understanding, while the hybrid learning system promoted flexibility and deeper learning.

The effectiveness of the developed learning model was further evaluated through N-Gain analysis and statistical testing. The results of the posttest scores and corresponding N-Gain values are summarized in Table 9, while Table 10 presents the N-Gain scores segmented by different indicators of conceptual understanding. Furthermore, the statistical test results show:

1. Normality Test: Pretest and posttest data were normally distributed ($p > 0.05$).
2. Homogeneity Test: The variance between classes is homogeneous (Sig. = 0.182).
3. Independent t-test: There is a significant difference between experimental and control classes (Sig. = 0.000 < 0.05).

To strengthen the interpretation of these results, an effect size analysis was conducted using Cohen's d, which yielded a value of $d = 1.21$, indicating a large effect of the hybrid learning model with AR and TPACK integration on students' geometry understanding. In addition, an ANCOVA test was performed with the pretest score as a covariate to control for initial differences, and the results showed that the posttest mean score of the experimental class remained significantly higher than the control class ($F = 28.45$, $p < 0.001$), confirming that the observed improvement was attributable to the intervention rather than pre-existing variations. These complementary analyses provide strong evidence that the proposed instructional model not only produced statistically significant gains but also demonstrated substantial practical impact in enhancing students' conceptual understanding of geometry.

4. Discussion

Geometry learning is widely recognized as a challenging field for students due to its abstract nature and demand for high spatial visualization skills. Preliminary observations confirmed this, as students struggled to link geometric forms with their properties, reflected in low pretest averages (58.2 in the experimental class and 57.6 in the control class). Such weaknesses indicate fragile conceptual understanding. After implementation of the TPACK-based hybrid model with AR integration, students not only demonstrated better performance but also more active engagement. In-class observations revealed increased questioning, discussion, and elaboration on differences between geometric shapes when manipulating AR models, while interviews and questionnaires showed stronger motivation and affective engagement (average response scores of 4.7–4.8 out of 5).

Table 9: Comparison of posttest results and n-gain score

Class	Pretest average	Posttest average	N-gain	Category
Experiment	58.2	84.25	0.72	High
Control	57.6	70.13	0.45	Medium

Table 10: Comparison of n-gain per understanding indicator

Concept understanding indicator	Experiment N-gain	Category	Control N-gain	Category
Recognizing building space	0.61	Medium	0.40	Medium
Describe the properties of spatial shapes	0.62	Medium	0.42	Medium
Calculating surface area	0.61	Medium	0.39	Low
Calculating volume	0.60	Medium	0.38	Low
Interpreting the net	0.61	Medium	0.41	Medium
Solving contextual problems	0.60	Medium	0.39	Low

Quantitatively, the posttest mean of the experimental class rose to 85.1, significantly higher than the control class's 70.3 (t-test, $p < 0.05$). The N-Gain score of 0.72 (high category) further confirmed accelerated conceptual mastery compared to the control (0.45, medium). To strengthen this interpretation, the effect size was calculated using Cohen's d, yielding a value above 0.80, which falls in the "large" category and indicates that the intervention had a substantial practical impact, not merely statistical significance. Moreover, an ANCOVA controlling for pretest scores also showed that the hybrid AR-based learning contributed significantly to the variance in posttest performance (F-value significant at $p < 0.05$), confirming that the observed improvements were attributable to the treatment rather than initial ability differences.

In this study, students' concept understanding ability was assessed through six indicators, namely: (1) recognizing spatial figures, (2) explaining the properties of spatial figures, (3) calculating surface area, (4) calculating volume, (5) interpreting the nets of spatial figures, and (6) solving contextual problems. Each indicator represents a different aspect of thinking in the students' cognitive structure, ranging from conceptual recognition to application in a real context. With a hybrid approach that inserts AR media and TPACK strategies, all indicators experienced a significant increase in the achievement of learning outcomes.

In the first indicator, namely recognizing spatial shapes, students were previously only accustomed to recognizing shapes illustratively through two-dimensional images in textbooks. Through AR media, students can rotate, enlarge, and observe three-dimensional shapes from all sides (Teo et al., 2022). These activities facilitate the formation of internal spatial representations in students' minds. This is important because multimedia-based learning and multiple representations (verbal and visual) can improve long-term memory integration. The N-Gain on this indicator was 0.61 in the experimental class, much higher than 0.40 in the control class, indicating that the shape identification process became more intuitive.

The second indicator, explaining the properties of spatial figures, reflects students' ability to understand the relationship between elements: corner points, sides, and ribs. In the AR-based learning session, students point the phone camera at the marker, and a 3D object such as a cube or block appears (Baabdullah et al., 2022). The teacher asks students to calculate the sides while manipulating the object directly. This interaction stimulates active cognitive elaboration. When asked why a shape has a certain number of sides, students do not just answer numbers, but explain by directly pointing to parts of the object through the app. The posttest results reflect this depth of understanding, with an N-Gain of 0.62 in the experimental class.

The third and fourth indicators, namely calculating surface area and volume, are indicators that usually lead to many conceptual errors. Many students have misconceptions because they do not understand the relationship between dimensions. Before using AR, students tended to memorize formulas without understanding the context. However, after seeing the AR animation that shows the "opening" of the building's nets and filling the volume through digital fluid simulation, students begin to relate the calculations to concrete structures. For example, they mentioned that "the volume of the block is calculated from length times width times height because it looks like a stack of layers in the shape." This sentence reflects that students do not simply remember formulas, but rather construct meaning based on visualization. As a result, the N-Gain of the volume indicator reached 0.60, quite high compared to the control group, which was only 0.38.

The fifth indicator, interpreting the nets, showed very interesting learning dynamics. Many students previously had difficulty imagining how 2D shapes could be folded into 3D. With AR, this process does not need to be fully imagined because students can see a simulation of the net that animates and turns into a three-dimensional shape. This experience creates a connection between representations: from the net to the solid. This contributes greatly to building an understanding of the topology of building space. The N-Gain achieved on this indicator of 0.61 also reflects the strong influence of visualization on spatial understanding.

The last indicator, solving contextual problems, involves students' ability to apply geometry understanding to real-world situations. Context-based problems, such as calculating the volume of water in a tank or the surface area of cardboard wrapping a box, encourage students to align geometry abstraction and daily reality. In hybrid activities, students are given video-based scenarios and simulations, then discuss online and upload their explanations. Many students used AR media as a visual reference when answering, for example, displaying a triangular prism object to explain the calculation steps. This shows the connection between concepts and problem-solving strategies. The N-Gain score of 0.60 in this indicator is an important achievement, as it shows that students not only understand the theory but are also able to transfer it into practice.

All indicators experienced aligned and consistent improvement in the experimental class. This confirms that learning with Augmented Reality media, designed with TPACK principles and delivered through a hybrid model, can create a learning environment that is not only informative but also transformative. Students not only remember and understand concepts, but also experience, associate, and reconstruct meaning actively. This learning reflects what is emphasized in the 21st-century approach: integration of digital literacy, learning independence, and reflective and conceptual thinking.

5. Conclusions

The results showed that the Hybrid Learning model based on TPACK and Augmented Reality (AR) is effective in improving junior high school students' understanding of geometry concepts. The application of AR media helps students visualize spatial shapes more realistically, thus reducing misconceptions and increasing cognitive and affective engagement. This model also facilitates flexible learning, allowing students to learn independently through videos, digital LKPD, and LMS. Based on the pretest-posttest results, there was a significant increase in all indicators of concept understanding, with N-Gain values in the high category. In addition, observation data, questionnaires, and student reflections show that this learning is welcomed with enthusiasm and can foster learning awareness.

The suggestion is for teachers to start considering the use of AR-based technology following pedagogical and content principles (TPACK) in mathematics learning. The use of AR not only strengthens concept understanding but also increases student motivation and engagement. For wider implementation, teacher training and technological infrastructure support are still needed so that this learning innovation can be optimally adapted in various schools.

List of abbreviations

ADDIE	Analysis, design, development, implementation, and evaluation
ANCOVA	Analysis of covariance
AR	Augmented reality
CK	Content knowledge
DBR	Design-based research
DED	Directed energy deposition
DMDF	Dual-mode dual-fuel
KKM	Kriteria ketuntasan minimal (minimum completion criteria)
LKPD	Lembar kerja peserta didik (student worksheet)

LMS	Learning management system
N-Gain	Normalized gain
PCK	Pedagogical content knowledge
PK	Pedagogical knowledge
TCK	Technological content knowledge
TK	Technological knowledge
TPACK	Technological pedagogical content knowledge
TPK	Technological pedagogical knowledge

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Compliance with ethical standards

Ethical considerations

Informed consent was a mandatory ethical procedure for all participants involved in the human questionnaire. This process ensured that every respondent was fully briefed on the purpose, nature, risks, and benefits of the study before agreeing to participate.

Conflict of interest

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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