Contents lists available at Science-Gate

International Journal of Advanced and Applied Sciences

Journal homepage: http://www.science-gate.com/IJAAS.html

Effect of homogeneous generalized thermoelasticity on semiconductor layer under magnetic field on Green and Naghdi model without energy dissipation

Mohamed H. Hendy [1, 2, *], Alaa Kamal Khamis [1]

[1]Department of Mathematics, Faculty of Science, Northern Border University, Arar, Saudi Arabia
[2]Department of Mathematics, Faculty of Science, Al Arish University, Al Arish, Egypt

ARTICLE INFO

Article history:
Received 9 September 2023
Received in revised form
14 January 2024
Accepted 28 January 2024

Keywords:
Thermo viscoelasticity
GN II model
Energy dissipation
Normal mode analysis
State-space method
Three-dimensional medium

ABSTRACT

This paper aims to explore the impact of viscosity and time on the spread of thermoelastic waves within a uniform and isotropic three-dimensional medium subject to a thermal load on its surface. This study utilizes the temperature-rate-dependent thermoelasticity based on the GN model, specifically applying the GN II model of generalized thermoelasticity, which does not account for energy dissipation. The normal mode analysis technique is employed to address the non-dimensional coupled field equations, yielding precise formulas for displacement, stress, temperature distribution, and strain. This issue is further illustrated by graphically depicting the field variables for a material similar to copper alongside the corresponding results. Comparative analyses of numerical data, with and without considering viscosity effects, suggest that the wave propagation speed will be limited.

1. Introduction

Two phenomena predicted by the classical uncoupled theory of thermoelasticity are incompatible with physical observations. First, there is no elastic term in this theory's heat conduction equation; second, the heat conduction equation is of the parabolic type, allowing heat waves to propagate at infinite speeds. Lord and Shulman (1967) developed a mathematical model of generalized thermoelasticity that is referred to as the 'LS model.' Biot (1956) introduced the classical dynamical-coupled theory. This classical linear theory of heat conduction is based on Fourier's law of thermal flux. This theory eliminated the paradox of infinite speed of propagation of thermal disturbances. Green and Lindsay (1972) developed a new model. Fourier's law of heat conduction remains unchanged in this model, but the classical energy equation and the constitutive equation have been modified by the addition of temperature rate-dependent terms. Two relaxation time parameters have been introduced in the GL model. This model is frequently referred to as temperature-rate-dependent thermo-elasticity (TRDTE). First, there is no elastic term in this theory's heat conduction equation; second, the heat conduction equation is of the parabolic type, allowing heat waves to propagate at infinite speeds. The three theories of generalized thermoelasticity were studied by Green and Naghdi (1991, 1993). Hetnarski and Ignaczak (1993) examined five generalizations to the coupled theory and obtained a number of important analytic results. The main objective of these approaches is to advocate a theory in which the propagation of heat is modeled with a finite speed. The modified heat conduction equation in this theory is of hyperbolic type, ensuring finite speeds of propagation for heat and elastic waves. Dhaliwal and Sherief (1980) further extended this theory to general anisotropic media in the presence of heat sources. Sherief et al. (2002, 2015) conducted a number of studies on the Lord-Shulman theory. Because of the rapid development of polymer science and the plastics industry, as well as the widespread use of materials at high temperatures in modern technology, theoretical investigation and application in thermoviscoelastic materials have become a significant task for solid mechanics. Several studies (Ezzat et al., 2002; Abd-Alla et al., 2003; Abd-Alla and Abo-Dahab, 2009; Deswal and Kalkal, 2011) Investigated wave propagation in linear thermovisco-elastic and electro-magneto-thermoviscoelastic solids. Chandrasekharaiah

* Corresponding Author.
Email Address: hendy442003@yahoo.com (M. H. Hendy)
https://doi.org/10.21833/ijaas.2024.02.014
Corresponding author's ORCID profile:
https://orcid.org/0000-0003-1919-1647

(1996) studied one-dimensional waves in a half-space of a homogeneous and isotropic material within the frame of the linear theory of thermoelasticity without energy dissipation. Dhaliwal and Wang (1995) introduced a heat flux-dependent theory of thermoelasticity, where the heat flux figures among the thermo-dynamical variables, with an evolution equation associated with it.

A class of mixed initial boundary-value problems is defined for porous media. Hetnarski and Ignaczak (1996) investigated the propagation of soliton-like thermoelastic waves at low temperatures. Tang and Araki (1997) studied thermal relaxation and a one-dimensional non-Fourier heat wave propagation. Chandrasekharaiah (1998) presented a review of temperature-rate dependent thermoelasticity theory, the recently developed theory of thermoelasticity without energy dissipation, and the new thermoelasticity theory. Kumar et al. (2017) investigated the propagation of thermoelastic harmonic plane waves under the two-temperature theory with two relaxation times. The one-dimensional formulation allows a simple interpretation of the results and puts in evidence the wave nature of heat propagation. Yu et al. (2013) solved a one-dimensional problem in fractional order generalized electro-magneto-thermoelasticity. Yu et al. (2015) introduced a novel compact numerical method for solving the two-dimensional non-linear fractional reaction-sub diffusion equations. The importance of state-space analysis is recognized in fields where the time behavior of any physical process is of interest. The state-space approach is more general than the classical Laplace and Fourier transform theories. Consequently, state-space theory is applicable to all systems that can analyzed by integral transforms in time and is applicable to many systems for which transform theory breaks down. One can refer to Ezzat (1994, 2012) and Ezzat and Youssef (2010) for a survey of the state space approach in continuum mechanics. Abouelregal (2019) introduced a two-temperature thermoelastic model without energy dissipation, which included higher-order time derivatives and two phase-lags. Hendy et al. (2019) employed the state space approach to fractional two-dimensional problems of thermo-viscoelasticity order heat transfer. The one-dimensional model of the theory applied to Stokes' flow of unsteady incompressible fluid due to a moving flat plate in the presence of both heat sources and a transverse magnetic field. Aldawody et al. (2019) studied the Green Naghdi theory of thermomechanics of continua to derive a linear theory of MHD thermoelectric fluid with fractional order. This theory permits the propagation of thermal waves at finite speed. De Sciarra and Salerno (2014) studied thermodynamic functions in thermoelasticity without energy dissipation. Khamis et al. (2020) and Helmy et al. (2021). Studied the magneto thermoviscoelastic waves with Green-Naghdi theory in a homogeneous isotropic hollow cylinder. Hendy et al. (2019) solved a two-dimensional problem for thermoviscoelastic materials. See, also Amin et al. (2022) and El-Attar et al. (2022).

The goal of this paper is to use normal mode analysis to investigate the aforementioned three-dimensional problem, as well as to use the state-space approach to examine the effects of viscosity on a three-dimensional thermoelastic homogeneous isotropic half-space solid body that is assumed stress-free with a surface that has undergone a thermal shock. Based on the GN model, without energy dissipation, the formulation for generalized thermoelasticity was developed. To obtain the precise analytical formulas for the variables under investigation, the normal mode analysis and state-space technique were both applied. The findings of numerical calculations for a particular material and theoretical comparisons in the presence and absence of viscosity effects indicate that the speed of propagation will bounded.

2. Mathematical model

The equations of motion for a homogeneous isotropic thermally conducting viscoelastic material without body forces, the relations stress-strain-temperature and strain-displacement, and the heat induction equation in the absence of a heat source can all be represented as:

$$\zeta_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \tag{1}$$

$$S_{ij} = \lambda u_{z,z}\sigma_{ij} + 2\mu\zeta_{ij} - \gamma T\sigma_{ij} \tag{2}$$

$$S_{ij,j} = \rho\frac{\partial^2 u_i}{\partial t^2} \tag{3}$$

$$\Gamma\,\nabla^2 T = \rho C_E\frac{\partial^2 T}{\partial t^2} + \gamma T_O\frac{\partial^2 e}{\partial t^2} \tag{4}$$

where,

$$\lambda = \lambda_o\left(1 + \lambda_o\frac{\partial}{\partial t}\right),\ \ \mu = \mu_o\left(1 + \mu_o\frac{\partial}{\partial t}\right),\ \ \gamma = \gamma_o\left(1 + \gamma_o\frac{\partial}{\partial t}\right),$$
$$\gamma_e = (3\lambda_o + 2\mu_o)\beta_T,\quad \gamma_o = (3\lambda_o\gamma_e + 2\mu_o\mu_e)\frac{\beta_T}{\gamma_e},\quad i,j,k = x,y,z$$

where, S_{ij} stands for the components of the stress tensor, ζ_{ij} for the components of the strain tensor, λ, μ Lame's constants, $\gamma = (3\lambda + 2\mu)\beta_T$ is a material constant characteristic of the theory. β_T is the coefficient of linear thermal expansion, ρ is the density, σ_{xy} is Kronecker delta, u_i are the components of the displacement vector, C_E is the specific heat at constant strain, $(\Gamma > 0)$ is a material constant characteristic of the theory, T is the temperature change, T_o is the medium's presumed reference temperature where $|T/T_o| \ll 1, e = e_{zz}$ is the cubical dilatation, $\lambda_o, \mu_o, \gamma_o$ are viscoelastic constants.

3. Formulation of the problem

We consider a homogeneous isotropic solid material that is thermoviscoelastic and covers the space in three dimensions.

$\Omega = \{((x, y, z): 0 \leq x < \infty, -\infty < y < \infty, -\infty < z < \infty\}$.

It is in contact with a time-varying heat source on the surface x=0. Furthermore, it is assumed that the surface x=0 is assumed to be stress free. By employing the Cartesian coordinates (x, y, z), it is assumed that the initial conditions for all physical variables are homogeneous, and as a result, the displacement components have the form $u_i = (u, v, w)$. Consequently, the following are the governing equations:

$$\rho \ddot{u} = [(\lambda_e + 2\mu_e) + (\lambda_e\lambda_o + 2\mu_o \mu_e)\tfrac{\partial}{\partial t}]u_{,xx} + \mu_e\left(1 + \mu_o\tfrac{\partial}{\partial t}\right)(u_{,yy} + u_{,zz}) + [(\lambda_e + 2\mu_e) + (\lambda_e\lambda_o + 2\mu_o \mu_e)\tfrac{\partial}{\partial t}](v_{,xy} + w_{,xz}) - \gamma_e\left(1 + \gamma_o\tfrac{\partial}{\partial t}\right) T_{,x} \tag{5}$$

$$\rho \ddot{v} = [(\lambda_e + 2\mu_e) + (\lambda_e\lambda_o + 2\mu_o \mu_e)\tfrac{\partial}{\partial t}]v_{,yy} + \mu_e\left(1 + \mu_o\tfrac{\partial}{\partial t}\right)(v_{,zz} + v_{,xx}) + [(\lambda_e + 2\mu_e) + (\lambda_e\lambda_o + 2\mu_o \mu_e)\tfrac{\partial}{\partial t}](w_{,yz} + u_{,yx}) - \gamma_e\left(1 + \gamma_o\tfrac{\partial}{\partial t}\right) T_{,y} \tag{6}$$

$$\rho \ddot{w} = [(\lambda_e + 2\mu_e) + (\lambda_e\lambda_o + 2\mu_o \mu_e)\tfrac{\partial}{\partial t}]w_{,zz} + \mu_e\left(1 + \mu_o\tfrac{\partial}{\partial t}\right)(w_{,yy} + w_{,xx}) + [(\lambda_e + 2\mu_e) + (\lambda_e\lambda_o + 2\mu_o \mu_e)\tfrac{\partial}{\partial t}](v_{,zy} + u_{,zx}) - \gamma_e\left(1 + \gamma_o\tfrac{\partial}{\partial t}\right) T_{,z} \tag{7}$$

$$\Gamma\, \nabla^2 T = \rho C_E \ddot{T} + \gamma_e T_O\left(1 + \gamma_o\tfrac{\partial}{\partial t}\right)\ddot{e} \tag{8}$$

$$S_{xx} = 2\,\mu_e\left(1 + \mu_o\tfrac{\partial}{\partial t}\right)u_{,x} + \lambda_e\left(1 + \lambda_o\tfrac{\partial}{\partial t}\right)e - \gamma_e\left(1 + \gamma_o\tfrac{\partial}{\partial t}\right)T \tag{9}$$

$$S_{yy} = 2\,\mu_e\left(1 + \mu_o\tfrac{\partial}{\partial t}\right)v_{,y} + \lambda_e\left(1 + \lambda_o\tfrac{\partial}{\partial t}\right)e - \gamma_e\left(1 + \gamma_o\tfrac{\partial}{\partial t}\right)T \tag{10}$$

$$S_{zz} = 2\,\mu_e\left(1 + \mu_o\tfrac{\partial}{\partial t}\right)w_{,z} + \lambda_e\left(1 + \lambda_o\tfrac{\partial}{\partial t}\right)e - \gamma_e\left(1 + \gamma_o\tfrac{\partial}{\partial t}\right)T \tag{11}$$

$$S_{xy} = \mu_e\left(1 + \mu_o\tfrac{\partial}{\partial t}\right)(u_{,y} + v_{,x}) \tag{12}$$

$$S_{xz} = \mu_e\left(1 + \mu_o\tfrac{\partial}{\partial t}\right)(u_{,z} + w_{,x}) \tag{13}$$

$$S_{yz} = \mu_e\left(1 + \mu_o\tfrac{\partial}{\partial t}\right)(v_{,z} + w_{,y}) \tag{14}$$

where,

$$e = (u_{,x} + v_{,y} + w_{,z}). \tag{15}$$

Moreover, a superimposed dot signifies time-related differentiation, while a subscript comma indicates spatial derivatives. The following non-dimensional parameters are defined, using the non-dimensionalized equations above:

$$(x', y', z') = \frac{1}{K}(x, y, z), \quad (u', v', w') = \frac{\lambda_e + 2\mu_e}{\gamma_e T_o K}(u, v, w), \quad t' = \frac{C_1 t}{K}$$

$$(\lambda_o{}', \mu_o{}', \gamma_o{}') = \frac{C_1}{K}(\lambda_o, \mu_o, \gamma_o), \quad \varphi = \frac{T}{T_o}, \quad S'_{xy} = \frac{\chi_{xy}}{\gamma_e T_o}$$

where, K is some standard length. By including the aforementioned non-dimensional parameters in Eqs. 5–14 and utilizing the relation in Eq. 15, we obtain the following set of governing equations.

$$\ddot{u} = \alpha\left(1 + \mu_o\frac{\partial}{\partial t}\right)\nabla^2 u + \left[(1-\alpha) + \{\lambda_o(1-2\alpha) + \alpha\mu_o\}\frac{\partial}{\partial t}\right]e_{,x} - \left(1 + \gamma_o\frac{\partial}{\partial t}\right)\varphi_{,x}, \tag{16}$$

$$\ddot{v} = \alpha\left(1 + \mu_o\frac{\partial}{\partial t}\right)\nabla^2 v + \left[(1-\alpha) + \{\lambda_o(1-2\alpha) + \alpha\mu_o\}\frac{\partial}{\partial t}\right]e_{,y} - \left(1 + \gamma_o\frac{\partial}{\partial t}\right)\varphi_{,y}, \tag{17}$$

$$\ddot{w} = \alpha\left(1 + \mu_o\frac{\partial}{\partial t}\right)\nabla^2 w + \left[(1-\alpha) + \{\lambda_o(1-2\alpha) + \alpha\mu_o\}\frac{\partial}{\partial t}\right]e_{,z} - \left(1 + \gamma_o\frac{\partial}{\partial t}\right)\varphi_{,z}, \tag{18}$$

$$C_T^2\,\nabla^2\varphi = \ddot{\varphi} + \varepsilon\left(1 + \gamma_o\tfrac{\partial}{\partial t}\right)\ddot{e} \tag{19}$$

$$S_{xx} = 2\,\alpha\left(1 + \mu_o\tfrac{\partial}{\partial t}\right)u_{,x} + (1-2\alpha)\left(1 + \lambda_o\tfrac{\partial}{\partial t}\right)e - \left(1 + \gamma_o\tfrac{\partial}{\partial t}\right)\varphi, \tag{20}$$

$$S_{yy} = 2\,\alpha\left(1 + \mu_o\tfrac{\partial}{\partial t}\right)v_{,y} + (1-2\alpha)\left(1 + \lambda_o\tfrac{\partial}{\partial t}\right)e - \left(1 + \gamma_o\tfrac{\partial}{\partial t}\right)\varphi, \tag{21}$$

$$S_{zz} = 2\,\alpha\left(1 + \mu_o\tfrac{\partial}{\partial t}\right)w_{,z} + (1-2\alpha)\left(1 + \lambda_o\tfrac{\partial}{\partial t}\right)e - \left(1 + \gamma_o\tfrac{\partial}{\partial t}\right)\varphi, \tag{22}$$

$$S_{xy} = \alpha\left(1 + \mu_o\tfrac{\partial}{\partial t}\right)(u_{,y} + v_{,x}) \tag{23}$$

$$S_{xz} = \alpha\left(1 + \mu_o\tfrac{\partial}{\partial t}\right)(u_{,z} + w_{,x}) \tag{24}$$

$$S_{yz} = \alpha\left(1 + \mu_o\tfrac{\partial}{\partial t}\right)(v_{,z} + w_{,y}) \tag{25}$$

where, $\varepsilon = \gamma_e^2 T_o / [\rho C_E(\gamma_e + 2\mu_e)]$ is the dimensionless thermoelastic coupling parameter, $C_T = c_3 / c_1$ is the non-dimensional finite thermal wave speed of GN theory, $c_1 = \sqrt{(\gamma_e + 2\mu_e)/\rho}$ is the longitudinal wave speed, $c_3 = \sqrt{\Gamma/\rho C_E}$ is the finite thermal wave speed of G-N theory, and $\alpha = \mu_e(\gamma_e + 2\mu_e)$.

Using Eqs. 16, 17, and 18 to differentiate them with regard to x, y, and z, respectively, we obtain:

$$\ddot{e} = \left[1 + \{\lambda_o - 2\alpha(\lambda_o - \mu_o)\}\tfrac{\partial}{\partial t}\right]\nabla^2 e - \left(1 + \gamma_o\tfrac{\partial}{\partial t}\right)\nabla^2\varphi. \tag{26}$$

According to Dhaliwal and Sherief (1980), the invariant stress S will be taken to equal the mean value of the primary stresses S_{ij},

$$S = \tfrac{1}{3}(S_{xx} + S_{yy} + S_{zz}) \tag{27}$$

From Eqs. 20-22 and using Eqs. 15 and 27 we get:

$$S = \left[1 - \tfrac{4\alpha}{3} + \left\{(1-2\alpha)\lambda_o + \tfrac{2\alpha}{3}\mu_o\right\}\tfrac{\partial}{\partial t}\right]e - \left(1 + \gamma_o\tfrac{\partial}{\partial t}\right)\varphi. \tag{28}$$

4. Normal mode analysis

By using normal modes analysis, the physical variables can be decomposed.

$$[u, v, w, e, \varphi, S, S_{ij}](x, y, z, t) = [u^*, v^*, w^*, e^*, \varphi^*, S^*, S^*{}_{ij}](x)\exp[\omega t + i(ay + bz)] \tag{29}$$

where, $u^* \cong u(x, y, z, t)$, i is the imaginary unit, ω (Complex) is the time constant, and a, b are the wave number in the x and y direction respectively. Using 29 in the Eqs. 19, 26, 28, we have

$$\frac{d^2\varphi^*}{dx^2} = A_1\,\varphi^* + A_2 S^* \quad (30)$$

$$\frac{d^2\chi^*}{dx^2} = A_3\,\varphi^* + A_4 S^* \quad (31)$$

where,

$$A_1 = [a^2 + b^2 + \frac{\omega^2}{C_T^2} + \frac{\varepsilon\omega^2(1+\gamma_o\omega)^2}{\beta C_T^2}], \quad A_2 = \frac{\varepsilon\omega^2(1+\gamma_o\omega)}{\beta C_T^2}$$

$$A_3 = [\frac{\omega^2(1+\gamma_o\omega)}{d} - \frac{\omega^2(d-\beta)(1+\gamma_o\omega)\{\beta+\epsilon(1+\gamma_o\omega)^2\}}{d\beta C_T^2}$$

$$A_4 = [a^2 + b^2 + \frac{\omega^2}{d} - \frac{\varepsilon\omega^2(d-\beta)(1+\gamma_o\omega)^2}{d\beta C_T^2}]$$

$$d = [1 + \{\lambda_o - 2\alpha\,(\lambda_o - \mu_o)\}\omega], \qquad \beta = [1 - \frac{4\alpha}{3} + \left\{(1-2\alpha)\lambda_o + \frac{2\alpha}{3}\mu_o\right\}\omega]$$

From Eqs. 30, 31, φ^* and S^*, then,

$$\frac{d^2V}{dx^2} = A\,(\omega)V(x) \quad (32)$$

$$V(x) = \begin{bmatrix}\varphi^* \\ S^*\end{bmatrix}, \qquad A(\omega) = \begin{bmatrix}A_1 & A_2 \\ A_3 & A_4\end{bmatrix}$$

We assume the non-dimensional boundary conditions as follows for the stress-free surface, where x=0.

(i) The thermal boundary condition:

$$q_n + v\varphi(x,y,z,t) = r(x,y,z,t), \text{on } x = 0 \quad (33)$$

where, q_n is normal component of the heat flux vector, v is Biot's number and $r(0,y,z,t)$ represents the intensity of the applied heat sources on $x = 0$. All the physical quantities are assumed to be bounded as $x \to +\infty$. From Eq. 33, we use the generalized Fourier's law of heat conduction in the non-dimensional form, then,

$$q_n = -\frac{\partial\varphi}{\partial n} \quad (34)$$

From Eqs. 33, 34, and 29, we obtain:

$$v\varphi^*(x) - D\varphi^*(x) = r^* \quad on \quad x = 0 \quad (35)$$

where, $D = d/dx$

(ii) Mechanical boundary condition that the bounding plane to the surface $x = 0$ has no traction anywhere and by using Eq. 29, then,

$$S^*(0,y,z,t) = S^*_{xx}(0,y,z,t) = S^*_{yy}(0,y,z,t) = S^*_{zz}(0,y,z,t) = 0$$

$$S^*(0) = S^*_{xx}(0) = S^*_{yy}(0) = S^*_{zz}(0) = 0 \quad (36)$$

5. State space approach

Eq. 32, then (see Youssef (2010)):

$$V(x) = \exp\left[-\sqrt{A(\omega)x}\right]V(0) \quad (37)$$

where,

$$V(0) = \begin{bmatrix}\varphi_o^* \\ 0\end{bmatrix}. \quad (38)$$

From Eq. 36, Eq. 35, Eq. 38 and omitted the positive exponential part to obtain a bounded solution for large x, in the solution Eq. 37. Now, we shall first find the form of the matrix $\exp\left[-\sqrt{A(\omega)x}\right]$.

$$\lambda^2 - (A_1 + A_4)\lambda + (A_1A_4 - A_2A_3) = 0. \quad (39)$$

Then, the solution of Eq. 39 is:

$$\lambda_j = \frac{(A_1+A_4)+(-1)^j\sqrt{(A_1-A_4)^2+4A_2A_3)}}{2} \qquad j = 1, 2.$$

The spectral decomposition of the matrix $A(\omega)$ from Sherief (1993) is:

$$A(\omega) = \lambda_1 E + \lambda_2 F \quad (40)$$

where, E and F are the projectors of $A(\omega)$ (see Simmons (2003))

$$E + F = 1 \;, EF = FE = 0 \;, E = E^2 \;, F = F^2 \quad (41)$$

Now, $\sqrt{A(\omega)}$ has the same projectors as of $A(\omega)$ and if p_1, p_2 are the eigenvalues of $\sqrt{A(\omega)}$ then, $p_1 = \sqrt{\lambda_1}$, $p_2 = \sqrt{\lambda_2}$. Then, the spectral decomposition of the matrix $\sqrt{A(\omega)}$ are:

$$\sqrt{A(\omega)} = p_1 E + p_2 F \quad (42)$$

where,

$$E = \frac{1}{\lambda_1-\lambda_2}\begin{pmatrix}A_1-\lambda_2 & A_2 \\ A_3 & A_4-\lambda_2\end{pmatrix}, \qquad F = \frac{1}{\lambda_2-\lambda_1}\begin{pmatrix}A_1-\lambda_1 & A_2 \\ A_3 & A_4-\lambda_1\end{pmatrix}. \quad (43)$$

Thus,

$$A^*(\omega) = \sqrt{A(\omega)} = \frac{1}{p_1+p_2}\begin{pmatrix}A_1 + p_1p_2 & A_2 \\ A_3 & A_4 + p_1p_2\end{pmatrix}. \quad (44)$$

The Taylor series expansion of the matrix exponential in Eq. 37 has the form:

$$\exp\left[-\sqrt{A(\omega)x}\right] = \exp[-A^*(\omega)] = \frac{\sum_{n=0}^{\infty}[-A^*(\omega)]^n}{n!}. \quad (45)$$

Using the Cayley-Hamilton theorem, we can express $A^{*2}(\omega)$ and higher orders of the matrix $A^*(\omega)$in terms of I and $A^*(\omega)$where I is the second order unit matrix. Thus, the Taylor series in Eq. 45 can be reduced to:

$$\exp[-A^*(\omega)] = a_o(x)I + a_1(x)A^*(\omega). \quad (46)$$

By the Cayley-Hamilton theorem, the characteristic roots p_1 *and* p_2 of the matrix $A^*(\omega)$ must satisfy Eq. 46 and we get:

$$\exp[-p_1x] = a_o(x)I + a_1(x)p_1 \quad (47)$$

$$\exp[-p_2x] = a_o(x)I + a_1(x)p_2. \quad (48)$$

From Eqs. 47 and 48, we have

$$a_o(x) = \frac{p_1 \exp[-p_2 x] - p_2 \exp[-p_1 x]}{p_1 - p_2}, \quad a_1(x) = \frac{\exp[-p_1 x] - \exp[-p_2 x]}{p_1 - p_2}. \tag{49}$$

From Eqs. 46 and 49, we have:

$$\exp\left[\sqrt{A(\omega)}\right] = \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix} \tag{50}$$

where,

$$B_{11} = \frac{(\lambda_1 - A_1)\exp(-\sqrt{\lambda_2}\,x) - (\lambda_2 - A_1)\exp(-\sqrt{\lambda_1}\,x)}{(\lambda_1 - \lambda_2)} \tag{51}$$

$$B_{12} = \frac{A_2[\exp(-\sqrt{\lambda_1}\,x) - \exp(-\sqrt{\lambda_2}\,x)]}{(\lambda_1 - \lambda_2)}$$

$$B_{21} = \frac{A_3[\exp(-\sqrt{\lambda_1}\,x) - \exp(-\sqrt{\lambda_2}\,x)]}{(\lambda_1 - \lambda_2)}$$

$$B_{22} = \frac{(\lambda_1 - A_4)\exp(-\sqrt{\lambda_2}\,x) - (\lambda_2 - A_4)\exp(-\sqrt{\lambda_1}\,x)}{(\lambda_1 - \lambda_2)}. \tag{52}$$

Finally, the solution of Eq. 32 is written as

$$V(x) = A_{ik}\,V(0) \quad , \quad j,k = 1,2 \tag{53}$$

Hence, using Eqs. 38, 51, and 52 in 53, the field variables $\varphi^*(x)$ and $S^*(x)$ are

$$\varphi^*(x) = \varphi_1 \exp(-\sqrt{\lambda_1}\,x) - \varphi_2 \exp(-\sqrt{\lambda_2}\,x) \tag{54}$$

$$S^*(x) = \frac{A_3 \varphi_0^*}{(\lambda_1 - \lambda_2)}\left[\exp(-\sqrt{\lambda_1}\,x) - \exp(-\sqrt{\lambda_2}\,x)\right] \tag{55}$$

where, $\varphi_1 = \varphi_0^*(A_1 - \lambda_2)/(\lambda_1 - \lambda_2)$, $\varphi_2 = \varphi_0^*(A_1 - \lambda_1)/(\lambda_1 - \lambda_2)$.

By using Eq. 35 we have φ_0^* as

$$\varphi_0^* = \frac{r(\lambda_1 - \lambda_2)}{v(\lambda_1 - \lambda_2) + \sqrt{\lambda_1}(A_1 - \lambda_2) - \sqrt{\lambda_2}\,(A_1 - \lambda_1)}.$$

By using Eqs. 29, 54, and 55 with Eq. 28 we have:

$$e^*(x) = e_1 \exp(-\sqrt{\lambda_1}\,x) - e_2 \exp(-\sqrt{\lambda_2}\,x) \tag{56}$$

where,

$$e_1 = \frac{\varphi_0^* A_3 + (A_1 - \lambda_2)(1 + \gamma_o \omega)}{\beta(\lambda_1 - \lambda_2)},$$

$$e_2 = \frac{\varphi_0^* A_3 + (A_1 - \lambda_1)(1 + \gamma_o \omega)}{\beta(\lambda_1 - \lambda_2)}.$$

Form Eq. 16, Eqs. 29, 54, and 56 in the Eq. 16, we have the displacement component $u^*(x)$ as:

$$(D^2 - \lambda_u^2)u^*(x) = \sum_{j=1}^{2}(-1)^{j-1} u_j (\lambda_j^2 - \lambda_u^2) \exp\left(-\sqrt{\lambda_j}\,x\right) \tag{57}$$

where,

$$\lambda_u^2 = a^2 + b^2 + \frac{\omega^2}{\alpha(1 + \mu_o \omega)},$$

$$u_j = \frac{\sqrt{\lambda_j}\left[e_j\{(1-\alpha) + \omega(\lambda_o(1 - 2\alpha) + \mu_o \alpha)\} - \varphi_j(1 + \gamma_o \omega)\right]}{\alpha(1 + \mu_o \omega)(\lambda_j^2 - \lambda_u^2)} \quad \text{j=1, 2.}$$

The general solution of the ordinary differential Eq. 57 is:

$$u^*(x) = u_0 \exp(-\lambda_u x) + \sum_{j=1}^{2}(-1)^{j-1} u_j \exp(-\sqrt{\lambda_j}\,x) \tag{58}$$

where, $\lambda_1^2 \neq \lambda_2^2 \neq \lambda_u^2$, u_0 is a constant.

From the boundary conditions Eq. 36. Using Eqs. 29, 54, 56, and 58 in the Eq. 20, then the stress components $\chi_{xx}^*(x)$ is:

$$S_{xx}^*(x) = S_3 \exp(-\lambda_u x) + \sum_{j=1}^{2}(-1)^{j-1} \chi_j \exp\left(-\sqrt{\lambda_j}\,x\right) \tag{59}$$

where,

$$S_3 = -2\alpha\lambda_u u_0(1 + \mu_0 \omega)$$

$$S_j = -2\alpha\sqrt{\lambda_j}\,u_j\,(1 + \mu_0 \omega) + e_j(1 - 2\alpha)(1 + \lambda_0 \omega) - \varphi_j(1 + \gamma_0 \omega), \text{ j =1, 2.}$$

From Eq. 36 and Eq. 59, we obtain u_0 as:

$$u_0 = \frac{\chi_1 - \chi_2}{2\alpha\lambda_u(1 + \mu_0 \omega)}.$$

6. Numerical example and discussions

Since we have $\omega = \omega_o + i\psi$, where i is the imaginary unit, $\exp(\omega t) = exp(\omega_o)\ (cos\ \psi t + i\ sin \psi t)$ and for small values of time, we can take $\omega = \omega_o$ (real). We will compute these physical variables numerically for a specific model in order to discuss the type of dependence of these physical variables on viscosity. The relevant numerical values for a material that resembles copper that we selected for this purpose are shown in Table 1.

Table 1: Numerical values

λ_e	$= 7.76 \times 1010 N/m^2$	μ_o	$= 0.09\ s$	ω	= 3
μ_e	$= 3.86 \times 1010\ N/m^2$	T_o	$= 293\ K$	a	=1.2
λ_o	$= 0.06\ s$	ε	= 0.0168	b	=1.3
β_T	$= 1.78 \times 10^{-5} K^{-1}$	α	= 0.25	v	=50
C_E	$= 383.1\ m^2/K$	r^*	=100	C_T	=2
ρ	$= 8954\ kg/m^3$				

From Table 1, the variations of the temperature distribution φ, the mean stress S, the displacement component u, and the stress component S_{xx} along x axis at two different plane $y = z = 0.$ and $y = z = 0.4$ for a particular time instant $t = 0.25$ have been shown for:

(iii) at $y = z = 0.$, $t = 0.25$, (solid by solid line)
(iv) at $y = z = 0.4$, $t = 0.25$,(solid by solid-dot line)
(v) at $y = z = 0.$, $t = 0.25$, (solid by solid-dot (bold) line)
(vi) at $y = z = 0.4$, $t = 0.25$, (solid by dashed line)

These variations are shown in Figs. 1-7

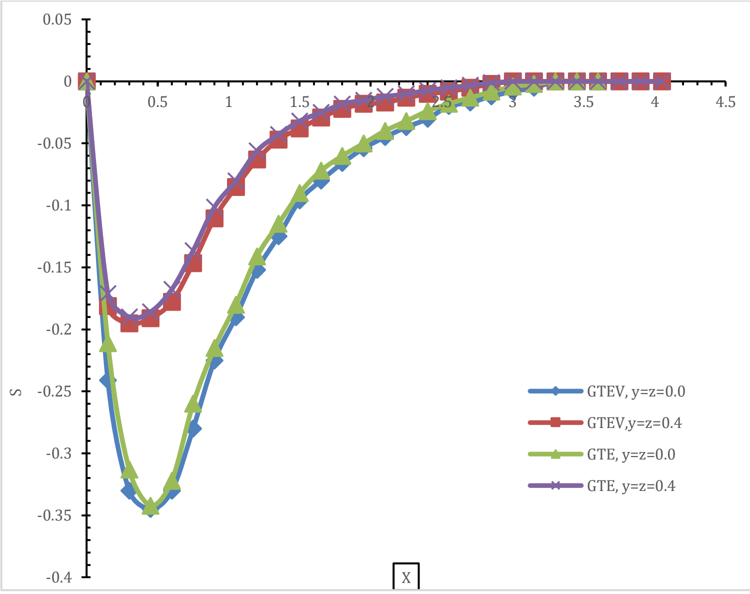

Fig. 1: Mean stress distribution S. vs. x at t=0.25

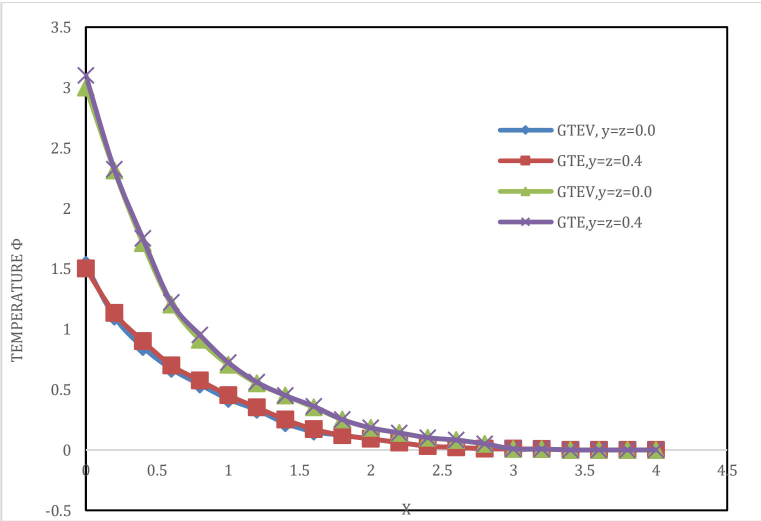

Fig. 2: The variation of temperature vs. distance x at t = 0.25

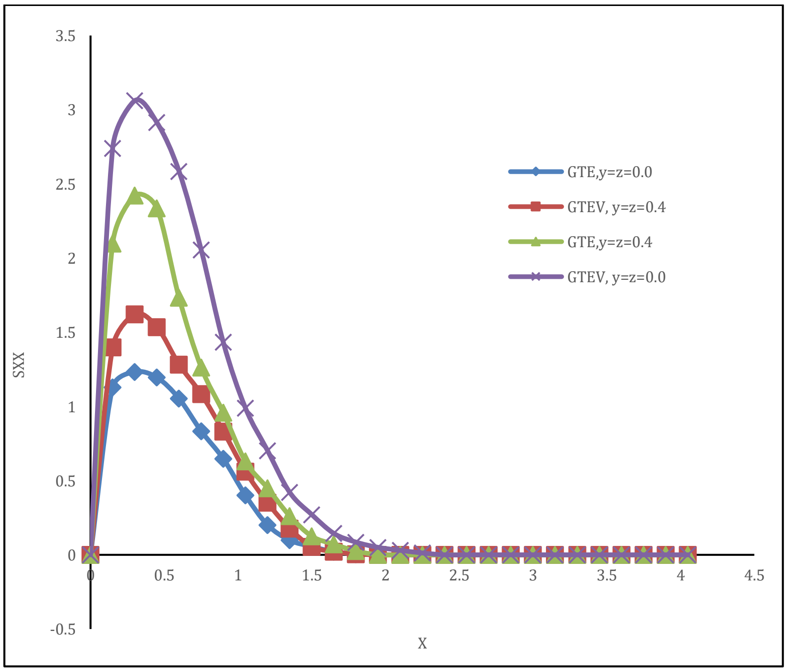

Fig. 3: Stress distribution vs. distance x at t=0.25

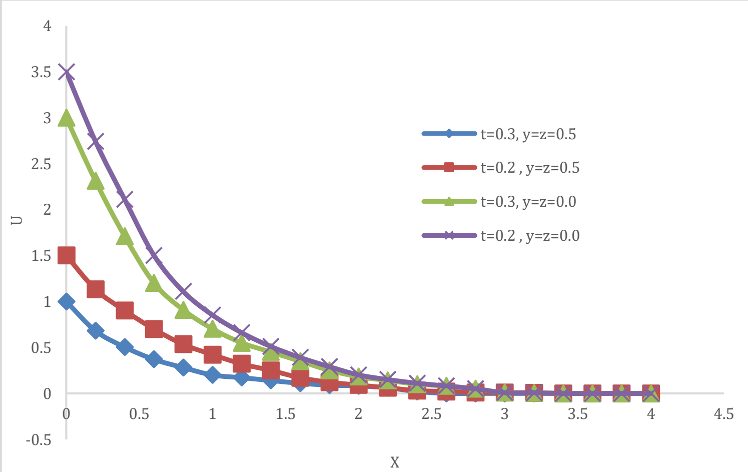

Fig. 4: Displacement distribution u and x at t =0.25

From Figs. 1-4 show that y *and* z decreasing effect on φ, S, u and S_{xx} for both GTVE and GTE model for fixed t. Also, it is depicted that the numerical values of S, u and S_{xx} are greater in GTVE model than GTE model for fixed, y, z *and* t, but in Fig. 2 the viscosity has no significant effect on φ. The maximum value of all the physical quantities attain in the case of GTVE at the plane $y = z = 0$.

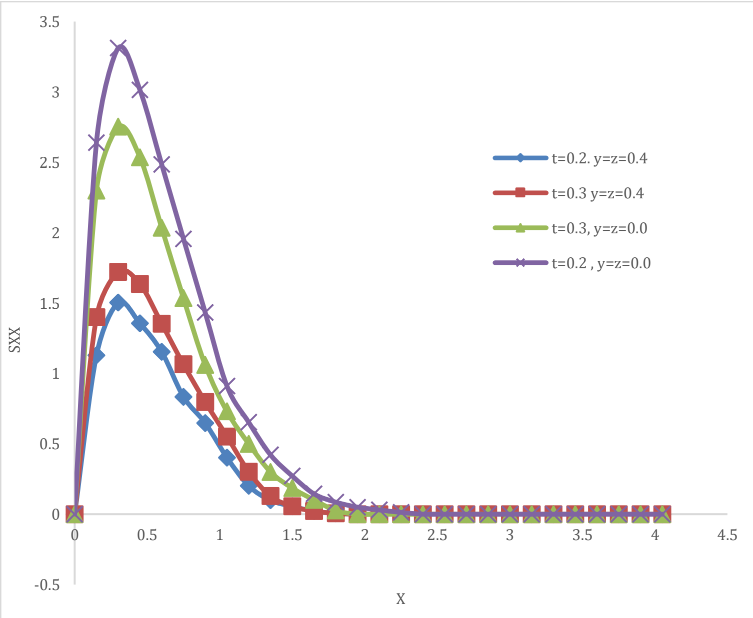

Fig. 5: Stress distribution S_{xx} vs. x for two time instants

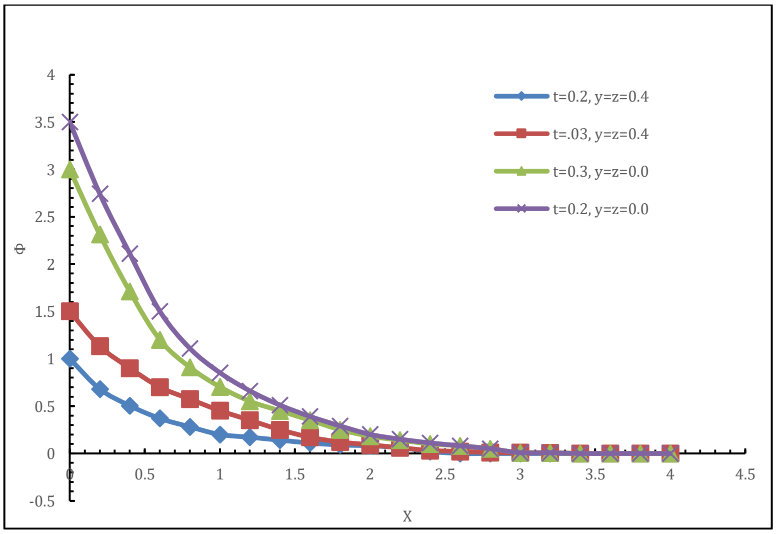

Fig. 6: Temperature distribution φ vs. x for two time instants

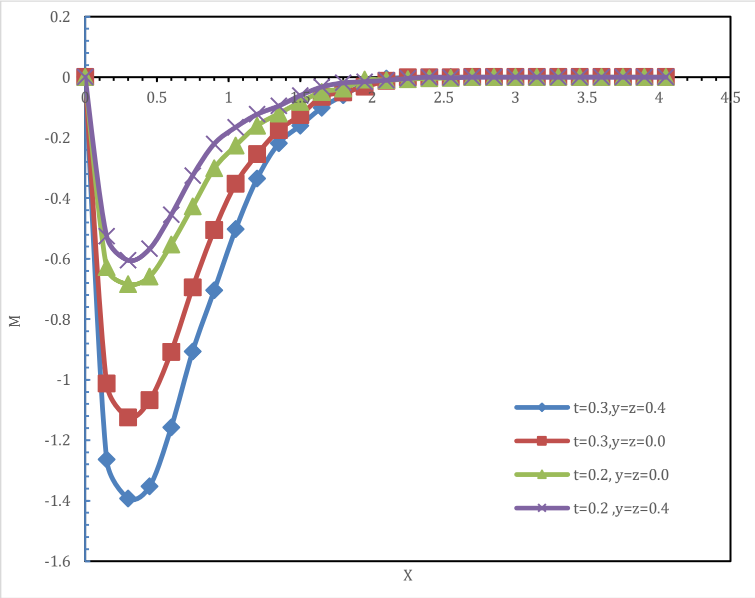

Fig. 7: Displacement distribution u vs. x for two time instants

Figs. 5-7 show the distributions of u, φ *and* S_{xx} for the GTVE model at y=z=0 and y=z=0.4 for two different time instants t=0.2 and t=0.3. Figs. 5-7 show that t has increasing influence on all the physical quantities.

Figs. 1-7 show that the wave propagation speeds of all physical quantities are finite and conform to the physical behavior of elastic materials. All of the additions to the illustration indicate that the boundary conditions Eqs. 35 and 36 are satisfied.

7. Conclusion

In this research, we apply the state space approach to solve a generalized thermo-elastic issue of an isotropic half-space with changing thermal conductivity.

- The numerical results presented here may be considered more general in the sense that they include the exact analysis of the Laplace transform domain of different field quantities. It is concluded from the graphical results presented in most cases that the Biot theory gives a set of results that intermediate those given by the GN theories.
- All of the distributions analyzed have a non-zero value only in a confined region of space; all values vanish in the same way, suggesting that the region has not yet experienced thermal disturbance. All physical variables' behavior at y=z=zero and y=z=0.4 are likely to be comparable, with minor magnitude variations.
- The behavior of physical processes is of interest, and the importance of state space analysis has been acknowledged (Bahar and Hetnarski, 1978).
- The state space method is more general than the traditional Laplace and Fourier transform methods. As a result, state space is applicable to all systems that can examined by integral transforms in time, as well as many systems where transform fails (Ogata, 1967).
- We attempted to implement a very useful technique in order to solve a three-dimensional generalized thermo-viscoelastic issue. Comparisons were made within the theory in the presence and absence of viscosity effects.

List of symbols

ρ	*Density*
t	*Time*
C_E	*Specific heat at constant strain*
B_i	*Components of magnetic field strength*
q_n	*Components of heat flux vector*
μ_o	*Magnetic permeability*
σ_{xy}	*Kronecker delta*
S_{ij}	*Components of stress tensor*
ζ_{ij}	*Components of strain tensor*
u_i	*Components of displacement vector*
λ, μ	*Lame's constants*
θ	$T - T_0$
T_o	*Reference temperature is chosen so that* $\lvert T - T_0 \rvert / T_0 \ll 1$
β_T	*Coefficient of linear thermal expansion*
ε	$\gamma_e^2 T_o / [\rho C_E(\gamma_e + 2\mu_e)]$; *Thermoelastic coupling parameter*
γ	$(3\lambda + 2\mu)\beta_T$
α	$\mu_e(\gamma_e + 2\mu_e)$

Acknowledgment

The authors gratefully acknowledge the approval and support of this research study by the Grant No. SCIA-2022-11-1256 from the Deanship of Scientific Research in Northern Border University, Arar, KSA.

Compliance with ethical standards

Conflict of interest

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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