

The Newtonian heating effect on MHD free convective boundary layer flow of magnetic nanofluids past a moving inclined plate



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ABSTRACT

The effect of magnetic strength on the MHD free convection flow of nanofluids over a moving inclined plate with Newtonian heating is analyzed. The governing partial differential equations with Newtonian heating boundary conditions are transformed into a system of nonlinear coupled ordinary differential equations (ODEs) by using similarity transformations. The Keller Box method was used as a solution method for ODEs. The skin friction and Nusselt number are evaluated analytically as well as numerically in a tabular form. Numerical results for velocity and temperature are shown graphically for various parameters of interest, and the physics of the problem is well explored. The significant findings of this study are promoting an angle of an aligned magnetic field, magnetic strength parameter, the angle of inclination parameter, local Grashof number, the volume fraction of nanoparticles, and Newtonian heating parameter. The result shows that the moving inclined plate in the same direction increases the skin friction coefficient and reduces the Nusselt number. It is also observed that the velocity of moving an inclined plate with the flow is higher compared to the velocity of moving an inclined plate against the flow. The temperature of a moving inclined plate with the flow is decreased much quicker than the temperature of a moving inclined plate against the flow. The other noteworthy observation of this study demonstrates that the Nusselt number in the Newtonian heating parameter shows that Fe₃O₄-kerosene is better than Fe₃O₄-water.

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1. Introduction

Over the last two decades, theoretical research on the dynamics and heat transfer characteristics of nanofluids has increased tremendously. Recently, the study of nanotechnology based on nanofluids has received broad attention due to its wide-ranging applications in various engineering's and technologies. Nanofluids are potential heat transfer fluids with enhanced thermophysical characteristics and heat transfer presentation applicable in many applications for improved performances. The term nanofluid was first proposed by [Choi and Eastman](#)

[\(1995\)](#) to describe a colloidal suspension with nanoparticles dispersed uniformly in a base fluid. Since then, rapid development related to nanofluids has been seen in many research papers that have been published. Heat transfer enhancement of nanofluids has been studied by [Xuan and Li \(2000\)](#), and they both further investigated the convective heat transfer and flow features of nanofluids. Moreover, [Wen and Ding \(2004\)](#) did an experimental investigation into the convective heat transfer of nanofluids under laminar flow conditions, while [Prasher et al. \(2006\)](#) measured the viscosity of the nanofluids and explored the nanofluids implication in thermal applications. A review and comparison of nanofluids' thermal conductivity and heat transfer enhancements have been reported by [Yu et al. \(2008\)](#). Improvement of the heat transfer in electronic cooling, heat exchangers, double plane windows, etc., is a tremendously important topic from the energy-saving point of view. Heat transfer analysis and fluid flow characteristics based on the

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Tiwari-Das model were examined by Sreedevi et al. (2021) and Reddy and Sreedevi (2021) for different effects and parameters. Magnetohydrodynamic (MHD) is the study of magnetic properties and the behavior of the electrically conducting fluid. In recent years, research on the topic of MHD has developed quickly by considering the different problems and situations (Ilias et al., 2016; 2017a; 2018; 2020). The magnetic field influences on flow and heat transfer have received the attention of researchers due to potential application in real-world problems. Magnetic nanofluids in this study are represented by ferrofluids, a liquid that becomes strongly magnetized in the presence of a magnetic field. Ferrofluids can be found in many potential fields, such as medicine, aerospace, science, and engineering. As pointed out by others, research on ferrofluids was initiated by Blums (2002), who investigated the heat and mass transfer behavior. Several advanced studies have addressed the ferrofluids heat transfer (Bozhko and Putin, 2003; Ganguly et al., 2004; Jue, 2006; Arulmurugan et al., 2006). The fluid flow and heat transfer in the presence of the magnetic field by Noranuar et al. (2021) found that the temperature increases while the nanofluid velocity reduces with a higher magnetic strength. In the year 2022, the study of the nanofluids with the presence of MHD effect by Rosaidi et al. (2022), Nayan et al. (2022), Bosli et al. (2022), and Ishak et al. (2022) found that with the effect of the magnetic field increase the heat transfer as well as the velocity of the study. Study on the MHD effect was discovered in different situations by some researchers such as Ilias (2018), Ahmad et al. (2019), Khashi'ie et al. (2019), and Soid et al. (2022).

Furthermore, an experiment of enhanced ferrofluid heat transfer under the influence of a magnetic field has been done by Lajvardi et al. (2010). Sheikholeslami and Rashidi (2015) found that the Nusselt number increases by analyzing ferrofluid heat transfer in the presence of the magnetic field. The flow of magnetic nanofluid over moving inclined surfaces occurs in many physical phenomena. Its importance can be seen in fluid where heat transfer is present. In the study of fluid flow over heated surfaces, the buoyancy forces exert a strong influence on the free convection flow field in the presence of a magnetic field. Ilias et al. (2017b) studied the influence of aligned and transverse magnetic fields on the two-dimensional natural convection boundary layer flow of a ferrofluid over a fixed vertical plate in the presence of convective boundary conditions. Two different base fluids (water and kerosene) containing magnetite (Fe_3O_4) as ferroparticles are considered. They found that the heat transfer rate at the plate surface with Fe_3O_4 -kerosene ferrofluid is higher than Fe_3O_4 -water. The industrial and technical applications of such problems include nuclear reactors cooled during an emergency shutdown, electronic devices cooled by fans, solar central receivers exposed to wind currents, and heat exchangers placed in a low-velocity environment.

The study based on free convection fluids by most researchers is because it is easier to analyze the behavior of magnetic nanofluids because it occurs naturally and without being influenced by external forces. Ilias et al. (2020) investigated unsteady aligned MHD boundary layer flow and heat transfer of magnetic nanofluids. They discovered that the free convection parameter known as the Grashof number increases the velocity as well as the heat transfer of their study. Rosaidi et al. (2022) investigated the behavior of MHD-free convection flow of magnetic nanofluids. The study found that the Grashof number has improved the velocity field and lowered the momentum boundary layer thickness. The study on free convection was rapidly established by considering the different problems and situations by Hamdan et al. (2020) and Mohamad et al. (2022). They found that the Grashof number has a significant impact on the velocity and momentum boundary layer thickness.

Newtonian heating has been widely applied in many types of research instead of constant surface temperature because of the failure of the constant temperature assumption adapted to physical situations. Newtonian heating is defined as the process of accentuating the surface resistance and desertion of internal resistance. Heat exchanger, conjugate heat transfers around fins, and the cooling mechanism for nuclear reactors are among the application that involves Newtonian heating in their processes. Four types of temperature distributions at the wall have been studied by Merkin (1994), and one of the temperature distributions is Newtonian heating. Studies on Newtonian heating have been conducted by many researchers, such as Singh and Makinde (2012). They investigated the presence of Newtonian heating in volumetric heat of MHD-free convection flow along with the inclined plate. At the same time, Uddin et al. (2012) studied the MHD free convective boundary layer flow of a nanofluid past a vertical plate with Newtonian heating. On the bright side, the research on Newtonian heating is still ongoing. For instance, Makinde (2013) analyzed the effects of viscous dissipation and Newtonian heating on the boundary layer flow of nanofluids over a flat plate, while Hayat et al. (2017) examined the Newtonian heating effect in nanofluids flow by a permeable cylinder. Apart from that, Ullah et al. (2017) investigated the effects of slip conditions and Newtonian heating on the MHD flow of Casson fluid over a nonlinearly stretching sheet saturated in a porous medium. Recently, Mohamed et al. (2019) investigated MHD slip flow and heat transfer on the stagnation point of a magnetite ferrofluid towards a stretching sheet with Newtonian heating. They found that the magnetite ferrofluid provided higher wall temperature and heat transfer capabilities compared to water. Yasin et al. (2019) studied the MHD stagnation flow in ferrofluid over a flat plate with Newtonian heating. The study found that the velocity is increased as well as the heat transfer in magnetic parameter. Aleem et al. (2020) investigated the MHD influence on different water-based nanofluids in a

porous medium with chemical reactions and Newtonian heating. They discovered that Ag-water nanofluid has a greater temperature due to its greater thermal conductivity value compared to others.

The study of the influence of magnetic field strength and Newtonian heating effects on nanofluid over the moving inclined plate has not been reported yet. The resulting governing equations are solved numerically using the Keller Box method for non-dimensional velocity and temperature profiles of the stationary inclined plate. Ilias (2018) studied the heat transfer rate of an MHD flow with a free convection effect by solving it by using the Keller box method. The study found that, for both unsteady and steady fluid flow cases, increasing nanoparticle volume fraction and magnetic field strength increase the Nusselt number. Further, the velocity and temperature profiles for moving the inclined plate with the flow and moving the inclined plate against the flow with the strength of the magnetic field are presented graphically.

2. Mathematical formulation

The steady two-dimensional, incompressible, laminar, hydromagnetic free convection of magnetic nanofluids flow with heat transfer over an inclined plate with the aligned and transverse magnetic field is considered. The plate is inclined at an angle of inclination γ measured in the clockwise direction and situated in an otherwise quiescent ambient fluid at temperature T_∞ . The gravitational acceleration g is acting downward. The physical coordinates (x, y) are chosen such that x – axis is measured along the plate, and the y – axis is measured normal to the surface of the plate, as shown in Fig. 1. Water and kerosene are used as the base fluids with magnetite (Fe_3O_4) as a nanoparticle. The base fluids and nanoparticles are in thermal equilibrium, and no-slip occurs between them. The spherical-shaped nanoparticles are considered. The viscous dissipation and radiation are ignored in the analysis.

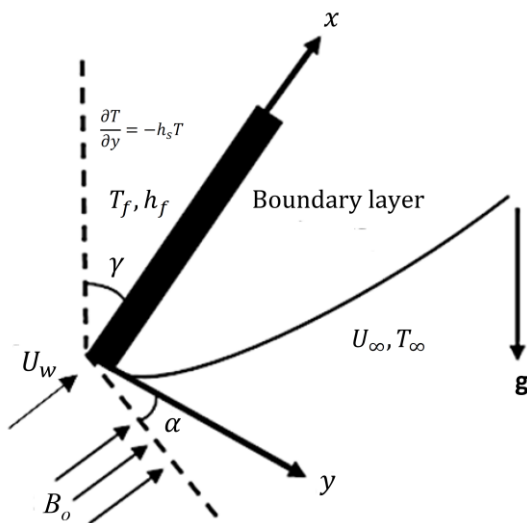


Fig. 1: Geometry of physical model

Under the above assumptions and following Tiwari and Das (2007), the equations of MHD boundary layer flow are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\mu_{nf}}{\rho_{nf}} \frac{\partial^2 u}{\partial y^2} + \frac{(\rho\beta)_{nf}}{\rho_{nf}} g \cos \gamma (T - T_\infty) - \frac{\sigma B^2(x)}{\rho_{nf}} \sin^2 \alpha (u - U_\infty) \tag{2}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_{nf} \frac{\partial^2 T}{\partial y^2} \tag{3}$$

The boundary conditions for the velocity and temperature of this problem are given by,

$$\begin{aligned} u(x, 0) = U(x) = \lambda U_\infty, \quad v(x, 0) = 0, \\ \frac{\partial T}{\partial y}(x, 0) = -h_s T, \quad u(x, \infty) = U_\infty, \quad T(x, \infty) = T_\infty \end{aligned} \tag{4}$$

where, u and v are the x (along the plate) and the y (normal to the plate) component of velocities, respectively. U_∞ is the free stream velocity, λU_∞ is the plate velocity, where λ is the moving inclined plate parameter. Moreover, the $h_s = h_0 c x^{\frac{n-1}{2}}$ represents the heat transfer parameter for Newtonian heating, T is the temperature of the nanofluids, and σ is the electrical conductivity. The transverse magnetic field assumed to be a function of the distance from the origin is defined as $B(x) = B_0 x^{\frac{1}{2}}$ with $B_0 \neq 0$, where x is the coordinate along the plate and B_0 is the magnetic field strength. The effective properties of nanofluids may be expressed in terms of the properties of base fluids, nanoparticles, and the volume fraction of solid nanoparticles as follows (Khan et al., 2015).

$$\begin{aligned} \rho_{nf} &= (1 - \phi)\rho_f + \phi\rho_s, \quad \mu_{nf} = \frac{\mu_f}{(1 - \phi)^{2.5}}, \\ (\rho C_p)_{nf} &= (1 - \phi)(\rho C_p)_f + \phi(\rho C_p)_s, \quad (\rho\beta)_{nf} = \\ &= (1 - \phi)(\rho\beta)_f + \phi(\rho\beta)_s \tag{5} \\ \alpha_{nf} &= \frac{k_{nf}}{(\rho C_p)_{nf}}, \quad \frac{k_{nf}}{k_f} = \frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + \phi(k_f - k_s)} \end{aligned}$$

where, ρ_{nf} is the effective density, ϕ is the solid volume fraction, ρ_f and ρ_s are the densities of pure fluid and nanoparticles, respectively. Moreover, μ_f is the dynamic viscosity of the base fluids, μ_{nf} is the effective dynamic viscosity, $(\rho C_p)_{nf}$ is the heat capacity of the nanofluids, $(\rho C_p)_f$ is specific heat parameters of the base fluids, $(\rho C_p)_s$ is the specific heat parameters of nanoparticles, $(\rho\beta)_{nf}$ is the thermal expansion coefficient, α_{nf} is the thermal diffusivity of the nanofluids, k_{nf} is the thermal conductivity of the nanofluids, k_f and k_s are thermal conductivities of the nanofluids and nanoparticles, respectively.

The continuity in Eq. 1 is satisfied by introducing a stream function $\psi(x, y)$ such as:

$$u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x} \tag{6}$$

The following similarity variables are introduced:

$$\eta = y \sqrt{\frac{U_\infty}{\nu_f x}} = \frac{y}{x} \sqrt{Re_x}, \quad \psi = \nu_f \sqrt{Re_x} f(\eta), \quad \theta = \frac{T - T_\infty}{T_\infty} \quad (7)$$

where, η is the similarity variable, $Re_x = U_\infty x / \nu_f$ is the Reynolds number, $f(\eta)$ the non-dimensional stream function and $\theta(\eta)$ the non-dimensional temperature.

By applying Eqs. 5 and 6, as well as Eqs. 7 and 2, and Eq. 3, we simplify these into a nonlinear system of ordinary differential equations:

$$f''' + (1 - \phi)^{2.5} \left(1 - \phi + \phi \left(\frac{\rho_s}{\rho_f} \right) \right) \frac{1}{2} f f'' + (1 - \phi)^{2.5} M (1 - f') \sin^2 \alpha + (1 - \phi)^{2.5} \left(1 - \phi + \phi \left(\frac{\rho \beta_s}{\rho \beta_f} \right) \right) Gr_x \theta \cos \gamma = 0 \quad (8)$$

$$\left(\frac{k_{nf}}{k_f} \right) \theta'' + \frac{Pr}{2} \left(1 - \phi + \phi \left(\frac{\rho C_p}{\rho C_p} \right) \right) f \theta' = 0 \quad (9)$$

subjected to the boundary conditions in Eq. 4, which becomes:

$$f(0) = 0, \quad f'(0) = \lambda, \quad \theta'(0) = -\omega(\theta(0) + 1) \\ f'(\eta) = 1, \quad \theta(\eta) = 0, \quad \text{as } \eta \rightarrow \infty \quad (10)$$

where, primes denote differentiation with respect to η , $\omega = h_s \sqrt{\frac{\nu_f x}{U_\infty}}$ is the Newtonian heating parameter, $M = \sigma B_0^2 / \rho U_\infty$ is the magnetic parameter, $Gr_x = g \beta_f T_\infty x / U_\infty^2$ is the local Grashof number and $Pr = (\mu C_p)_f / k_f$ is the Prantl number. Note that λ denotes the direction of motion of the plate with $\lambda = 0$ for the static plate, while $\lambda = 0.2$ and $\lambda = -0.2$ for the fourth and back motion of the plate, respectively. In order to have a true similarity solution, the parameter Gr_x must be constant and independent of x . This condition will be satisfied if the thermal expansion coefficient β_f proportional to x^{-1} . Hence, by assuming the work of Makinde (2011), $\beta_f = ax^{-1}$, where a is a constant but have the appropriate dimension. Substituting $\beta_f = ax^{-1}$ into the parameter Gr_x will result in $Gr = agT_\infty / U_\infty^2$. The quantities of engineering interest are the skin-friction coefficient, C_f at the surface of the plate and the local Nusselt number, Nu_x , which is defined as:

$$C_f = \frac{\tau_w}{\rho_f U_\infty^2}, \quad Nu_x = \frac{x q_w}{k_f (T - T_\infty)} \quad (11)$$

where, τ_w is the wall skin friction or shear stress at the plate and q_w is the heat flux from the plate, which is given by:

$$\tau_w = \mu_{nf} \left(\frac{\partial u}{\partial y} \right)_{y=0}, \quad q_w = -k_{nf} \left(\frac{\partial T}{\partial y} \right)_{y=0} \quad (12)$$

Substituting Eqs. 7 and 12 into Eq. 11, we obtain:

$$(Re_x)^{\frac{1}{2}} C_f = \frac{1}{(1 - \phi)^{2.5}} f''(0), \quad \frac{Nu_x}{(Re_x)^{\frac{1}{2}}} = -\frac{k_{nf}}{k_f} \frac{\theta'(0)}{\theta(0)} \quad (13)$$

3. Numerical solution

Eqs. 8 and 9, subject to the boundary conditions in Eq. 10 are solved numerically using the Keller-box method as described in the books of Na (1979) and Cebeci and Bradshaw (2012). The solution is achieved through four stages:

1. Transform Eqs. 8 and 9 into a first-order system.
2. Formulate the difference equations using central differences.
3. Use Newton's method to linearize the resulting algebraic equations and express them in matrix-vector format.
4. Apply the block tridiagonal elimination method to solve the linear system.

4. Result and discussion

In order to look into the physical insight of the problem, the velocity and temperature profiles against the dimensionless position η , for both magnetic nanofluids have been discussed by assigning different numerical values to the parameter: Angle for the aligned magnetic field α , magnetic strength M , angle of inclination of the plate γ , local Grashof number Gr_x , the volume fraction of nanoparticles ϕ and Newtonian heating parameter ω and their effects on flow and heat transfer characteristics are analyzed graphically.

Two different base fluids are considered, namely kerosene and water with magnetic nanoparticle Fe₃O₄. Table 1 shows the thermophysical properties of kerosene, water, and Fe₃O₄. The value of the Prandtl number for water is taken as 6.2, while 21 is the Prandtl number for kerosene. The effect of solid volume fraction ϕ is investigated in the range of $0 \leq \phi \leq 0.20$, in which $\phi = 0$ signifies pure fluid water or kerosene. To validate the numerical method's accuracy, a direct comparison was made with the previously reported numerical results of Blasius (1908) and Khan et al. (2015) for Fe₃O₄-water and Fe₃O₄-kerosene with respect to aligned magnetic field parameters in the absence of free convection parameter. From Table 2, the present results are observed to be in good agreement with those of the previous findings.

The variation in velocity and temperature profiles for different values of inclined angle of the magnetic field, α for magnetic nanofluids is disclosed in Fig. 2. As the values of α increase, velocity profiles gradually increase, but an increase in the inclined angle of the magnetic field α loses the motion of the fluid, which causes a reduction in the temperature of the fluid. When $\alpha = 0^\circ$ it indicates that there is no magnetic field and because of the changes in the aligned field position of the magnetic field, it attracts the nanoparticles. It is noticed that a rise is detected in the momentum boundary layer while a decline is detected in the thermal boundary layer.

Table 1: Thermophysical properties of base fluids and nanoparticles (Mojumder et al., 2015; Sheikholeslami et al., 2015)

Physical properties	Water	Kerosene	Fe ₃ O ₄
$\rho(kg/m^3)$	997.1	780	5200
$C_p(J/kgK)$	4179	2090	670
$k(W/mK)$	0.613	0.149	6
$\beta \times 10^{-5}(K^{-1})$	21	99	1.3
Pr	6.2	21	

Table 2: Comparison of the skin friction coefficient for different values of volume fraction of nanoparticles

Volume fraction	Skin friction		
	$Gr_x = 0, \alpha = 90^\circ, \gamma = 0^\circ, M = 0$		
	Blasius (1908)	Khan et al. (2015)	Current study
Pure water	0	0.33206	0.332059
Fe ₃ O ₄ -water	0.01	-	0.34324
	0.05	-	-
	0.10	-	0.45131
	0.15	-	-
	0.20	-	0.59517
Pure kerosene	0	-	0.332059
Fe ₃ O ₄ -kerosene	0.01	-	0.34557
	0.05	-	-
	0.10	-	0.47336
	0.15	-	-
	0.20	-	0.63950

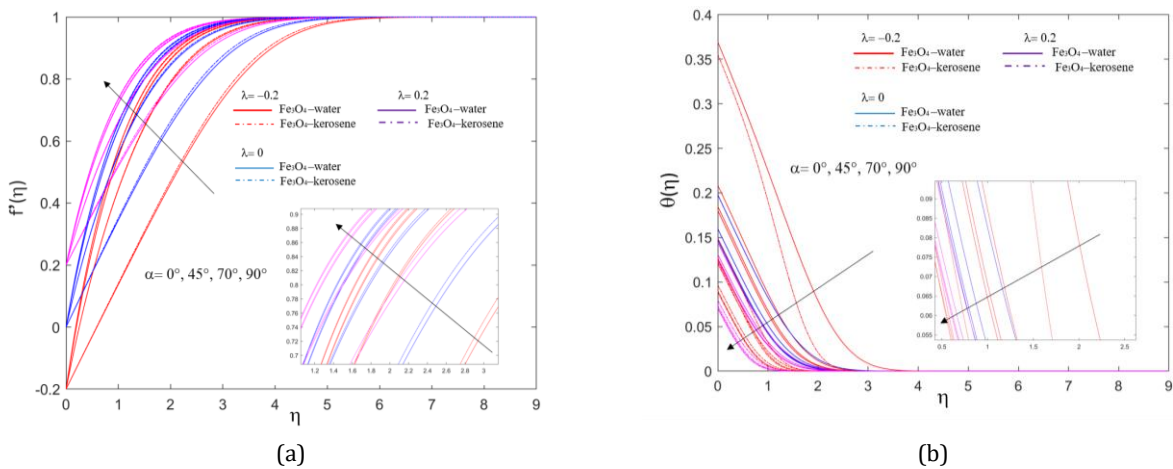


Fig. 2: Effect of aligned magnetic field parameters on the (a) velocity and (b) temperature profiles for $M = 1, \gamma = 45^\circ, Gr_x = 0.1, \varphi = 0.05, \text{ and } \omega = 0.1$

The effects of different values of the magnetic strength, $M = 0, 1, 2, \text{ and } 4$ of the dimensionless velocity and temperature profiles are presented in Fig. 3. From Figs. 3a and 3b, the velocity increases while the temperature decreases as the values of M increase. These results are similar to those reported by Bosli et al. (2023) and Ilias et al. (2020). From Figs. 3a and 3b indicated that Fe₃O₄-kerosene shows the highest velocity profiles and lowest temperature compared to Fe₃O₄-water. However, based on Table 3, the skin friction, which measures the drag exerted by a moving fluid on the plate surface, also increases. This is due to the fact that Newtonian heating at the plate surface causes a high surface temperature, which results in an increase in the buoyancy force. When $M = 0$, this indicates that there is no magnetic force. It is means by when magnetic field value increase, it pushes the fluid towards the plate and thus, the momentum boundary layer decreases. This situation leads to an increase in magnetic nanofluids' velocity near the plate surface by overplaying the effect of the Lorentz force. Fig. 4a shows the effect of the angle of inclination γ on velocity profile for both magnetic nanofluids. The velocity decreases for increasing values of the angle of inclination γ . The

fluid velocity is higher when the surface is vertical, $\gamma = 0^\circ$ than when it is inclined. This is because the angle of inclination decreases the effect of the buoyancy force due to the gravity component. It is obvious that the buoyancy force is maximum for $\gamma = 0^\circ$ and there is no buoyancy force for $\gamma = 90^\circ$, horizontal plate. The temperature profile can be noticed in Fig. 4b. It is directly proportional to the increase in inclination angle. As γ increases, the skin friction decreases slightly while the temperature of magnetic nanofluids increases considerably. The momentum boundary layer and thermal boundary layer thickness become thicker with the increasing inclination angle.

The effects of the Grashof number Gr_x on velocity and temperature profiles are presented in Fig. 5. The velocity increases as the Grashof number increases. This is due to the presence of the buoyancy effect, which enhances the velocity. Since the Gr_x is the ratio of the buoyant to the viscous force that acts on a fluid. Rising buoyancy forces allow viscosity to decrease, and with it, the boundary layer of momentum decreases continuously. On the other hand, the increasing Grashof number reduced the temperature profile for both magnetic nanofluids.

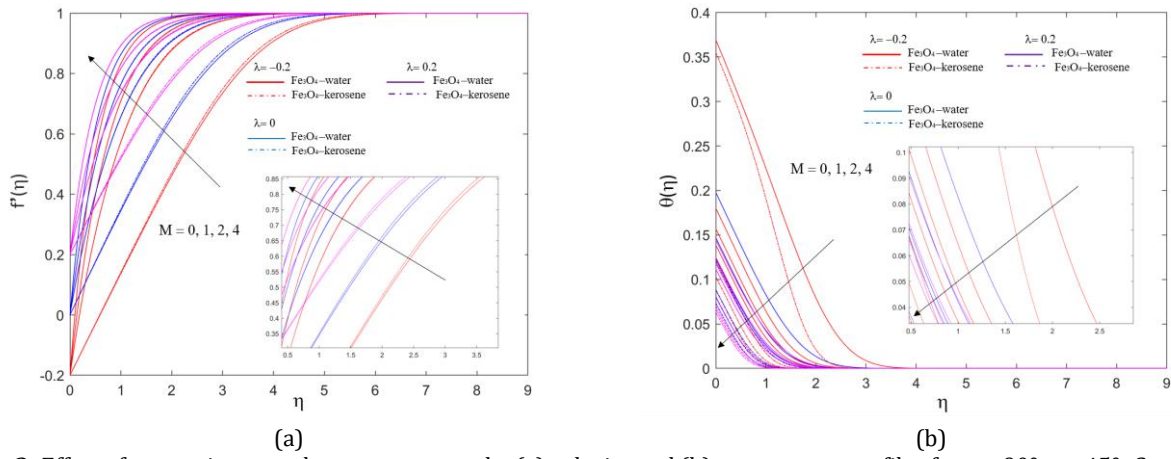


Fig. 3: Effect of magnetic strength parameters on the (a) velocity and (b) temperature profiles for $\alpha = 90^\circ$, $\gamma = 45^\circ$, $Gr_x = 0.1$, $\phi = 0.05$, and $\omega = 0.1$

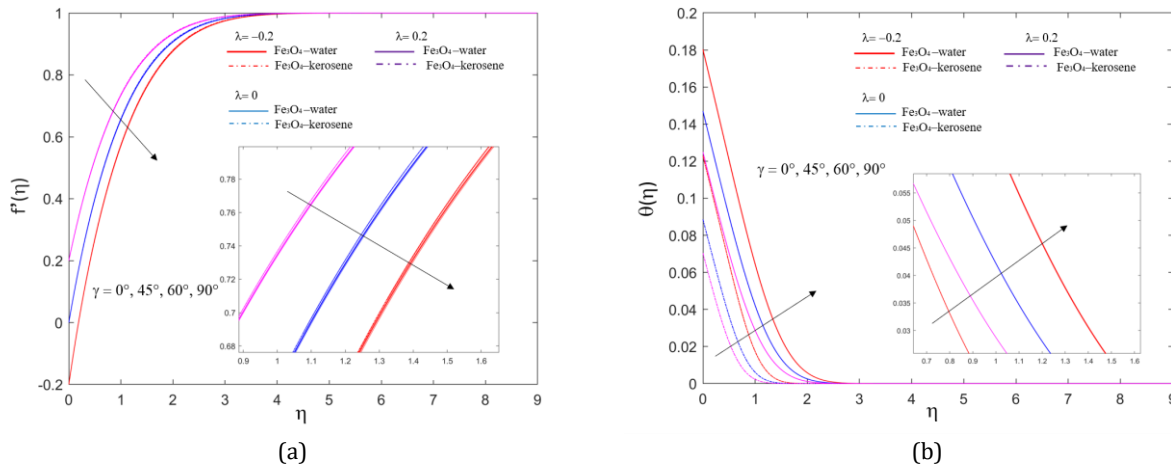


Fig. 4: Effect of inclined plate parameters on the (a) velocity and (b) temperature profiles for $\alpha = 90^\circ$, $M = 1$, $Gr_x = 0.1$, $\phi = 0.05$, and $\omega = 0.1$

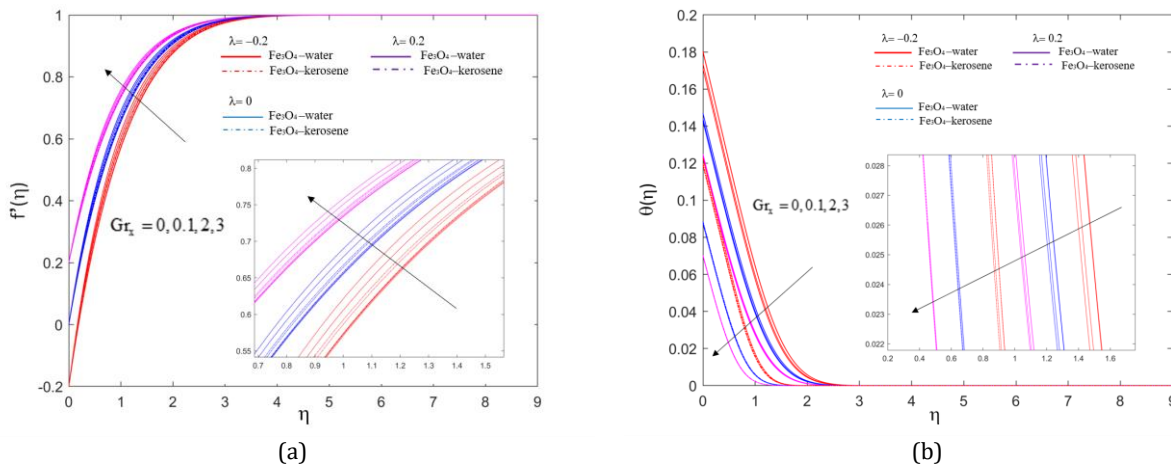


Fig. 5: Effect of local Grashof number on the (a) velocity and (b) temperature profiles for $\alpha = 90^\circ$, $M = 1$, $\gamma = 45^\circ$, $\phi = 0.05$, and $\omega = 0.1$

The dimensionless velocity for both magnetic nanofluids for different values of nanoparticle volume fraction is shown in Fig. 6. The velocity gets decelerated with increasing values of nanoparticles, but a converse action has been seen in the temperature profile. Increasing the volume fraction parameter results in an intensification of the temperature profile, in which the friction force is increased within the fluid. Therefore, it can be

concluded that the temperature can be controlled by varying the nanoparticles' volume fraction as the temperature profile is higher for $\phi = 0.20$. The enhancement of magnetic nanofluids' thermal conductivity is linked to the delicacy of the width of the thermal boundary layer by ϕ . To put it another way, the higher the thermal conductivity of a fluid, the higher the thermal diffusivity.

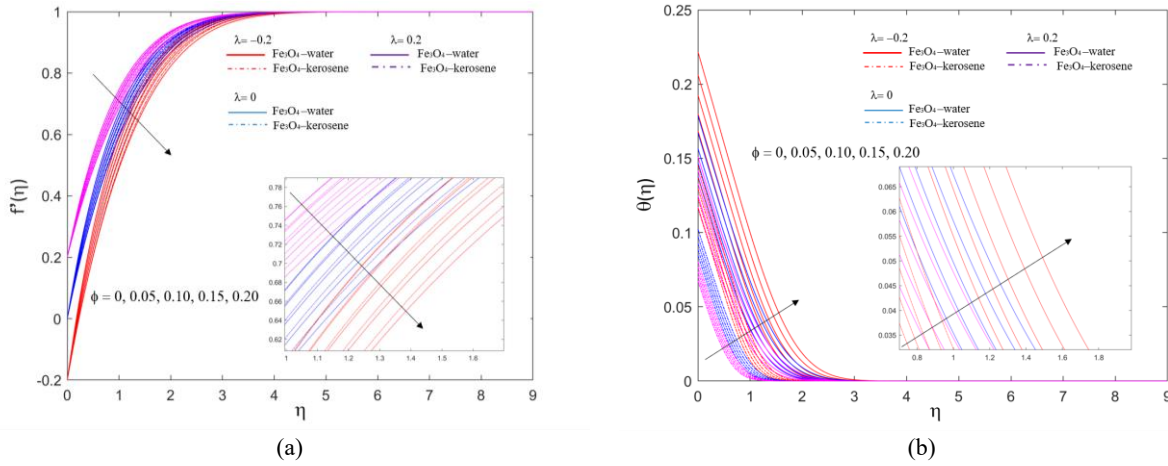


Fig. 6: Effect of volume fraction of nanoparticles on the (a) velocity and (b) temperature profiles for $\alpha = 90^\circ$, $M = 1$, $\gamma = 45^\circ$, $Gr_x = 0.1$, and $\omega = 0.1$

It is observed that an increase in the Newtonian heating parameter, ω , boosts the velocity profile as well as momentum boundary layer thickness (Fig. 7a). The behaviour of ω on temperature profile is analyzed in Fig. 7b. Obviously, temperature and

thermal boundary layer tend to decrease for a rise in Newtonian heating parameter. Newtonian heating expresses that the heat-transfer rate through a sidewall is proportional to the local sidewall temperature.

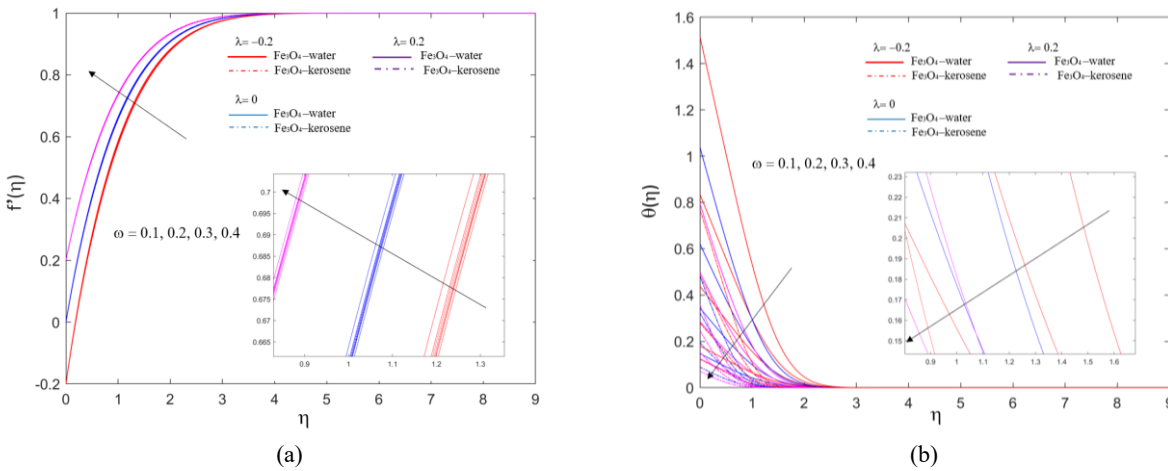


Fig. 7: Effect of Newtonian heating parameter on the (a) velocity and (b) temperature profiles for $\alpha = 90^\circ$, $M = 1$, $\gamma = 45^\circ$, $Gr_x = 0.1$, and $\phi = 0.05$

Based on Table 3 and Table 4, it was clearly stated the skin friction coefficient at the wall of the inclined plate is increasing in magnitude with an increase in aligned magnetic field α , magnetic strength M , Grashof number Gr_x , the volume fraction of nanoparticles ϕ , for both magnetic nanofluids and Newtonian heating parameters ω . The highest wall shear stress occurs when the magnetic strength of the moving inclined plate against the flow, $\lambda = -0.2$ increases. The increasing pattern for Nusselt number occurs for the same parameters of skin friction, and the highest rate of heat transfer occurs when the volume fraction of nanoparticles increases for moving inclined plate along the flow, $\lambda = 0.2$ for both Fe_3O_4 -water and Fe_3O_4 -kerosene.

5. Conclusion

Physically, aligned magnetic field parameter α , magnetic strength parameter M , the inclination of plate parameter γ , Grashof number Gr_x , the volume

fraction of nanoparticles ϕ and Newtonian heating parameter ω were studied in detail, and the results are discussed graphically.

From the results and discussion, the following specific conclusions for both magnetic nanofluids are obtained:

- The velocity of both magnetic nanofluids increases due to increasing α , M , Gr_x and ω .
- The velocity of both magnetic nanofluids decreases with the increase in γ and ϕ .
- The temperature of magnetic nanofluids decreases when the parameter of α , M , Gr_x , and ω are increasing.
- The temperature of magnetic nanofluids increases with the increase in γ and ϕ .
- The skin friction coefficient increases due to increasing α , M , Gr_x , ϕ and ω .
- The skin friction coefficient decreases due to increasing γ .

- The Nusselt number gets enhanced due to an increase in α, M, Gr_x, ϕ and ω .
- The Nusselt number decreases when γ increases.
- The velocity of both magnetic nanofluids increases faster for moving the inclined plate along the flow compared to moving the inclined plate against the flow due to an increase in M .
- The temperature of both magnetic nanofluids decreased faster for moving the inclined plate along the flow compared to moving the inclined plate against the flow due to an increase in M .

Table 3: Variation in skin friction coefficient and Nusselt number at different dimensionless parameters for Fe₃O₄-water

α	M	γ	Gr_x	ϕ	ω	Fe ₃ O ₄ -water					
						Skin friction			Nusselt number		
						$\lambda = -0.2$	$\lambda = 0$	$\lambda = 0.2$	$\lambda = -0.2$	$\lambda = 0$	$\lambda = 0.2$
0°						0.390526	0.401097	0.374142	0.413811	0.675797	0.884818
45°	1	45°	0.1	0.05	0.1	0.974740	0.842653	0.698469	0.646475	0.810865	0.964048
70°						1.253845	1.068085	0.873224	0.715651	0.859877	0.997307
90°						1.327643	1.128218	0.920244	0.731533	0.871496	1.005430
		0				0.390526	0.401097	0.374142	0.413811	0.675797	0.884818
	1	45°	0.1	0.05	0.1	1.327643	1.128218	0.920244	0.731533	0.871496	1.005430
						1.842597	1.551014	1.253387	0.823089	0.940569	1.055194
						2.581939	2.162619	1.739019	0.916707	1.014019	1.110244
		0°				1.330294	1.130201	0.921801	0.732065	0.871842	1.005665
	1	45°	0.1	0.05	0.1	1.327643	1.128218	0.920244	0.731533	0.871496	1.005430
						1.325764	1.126814	0.919143	0.731156	0.871251	1.005262
						1.321214	1.123420	0.916481	0.730241	0.870659	1.004859
		0				1.321214	1.123420	0.916481	0.730241	0.870659	1.004859
	1	45°	0.1	0.05	0.1	1.327643	1.128218	0.920244	0.731533	0.871496	1.005430
						1.443176	1.216506	0.990345	0.754149	0.886633	1.015938
						1.499742	1.260994	1.026239	0.764831	0.894076	1.021229
		0				1.237918	1.048399	0.852391	0.693366	0.825912	0.953219
		0.05				1.327643	1.128218	0.920244	0.731533	0.871496	1.005430
	1	45°	0.1	0.10	0.1	1.428831	1.218274	0.996807	0.768839	0.916723	1.057703
						1.543682	1.320526	1.083741	0.805206	0.961609	1.110141
						1.674988	1.437462	1.183160	0.840512	1.006141	1.162830
		0				1.327643	1.128218	0.920244	0.731533	0.871496	1.005430
	1	45°	0.1	0.05	0.2	1.336817	1.134641	0.925069	0.733371	0.872614	1.006161
						1.350900	1.143663	0.931474	0.736177	0.874180	1.007129
						1.374967	1.157212	0.940373	0.740933	0.876522	1.008471

Table 4: Variation of skin friction coefficient and Nusselt number at different dimensionless parameters for Fe₃O₄-kerosene

α	M	γ	Gr_x	ϕ	ω	Fe ₃ O ₄ -kerosene					
						Skin friction			Nusselt number		
						$\lambda = -0.2$	$\lambda = 0$	$\lambda = 0.2$	$\lambda = -0.2$	$\lambda = 0$	$\lambda = 0.2$
0°						0.394100	0.405806	0.380720	0.437261	1.064942	1.552865
45°	1	45°	0.1	0.05	0.1	0.974460	0.844177	0.701071	0.883899	1.300338	1.676167
70°						1.253463	1.069035	0.874977	1.017912	1.387825	1.730531
90°						1.327248	1.129064	0.921836	1.048754	1.408776	1.744046
		0				0.394100	0.405806	0.380720	0.437261	1.064942	1.552865
	1	45°	0.1	0.05	0.1	1.327248	1.129064	0.921836	1.048754	1.408776	1.744046
						1.842195	1.551431	1.254283	1.227722	1.535320	1.829001
						2.581597	2.162816	1.739513	1.414334	1.674562	1.927734
		0°				1.328657	1.129930	0.922436	1.049241	1.409014	1.744177
	1	45°	0.1	0.05	0.1	1.327248	1.129064	0.921836	1.048754	1.408776	1.744046
						1.326251	1.128452	0.921412	1.048409	1.408608	1.743953
						1.323837	1.126973	0.920388	1.047574	1.408201	1.743728
		0				1.323837	1.126973	0.920388	1.047574	1.408201	1.743728
	1	45°	0.1	0.05	0.1	1.327248	1.129064	0.921836	1.048754	1.408776	1.744046
						1.389741	1.168230	0.949173	1.070048	1.419457	1.750009
						1.421058	1.188434	0.963423	1.080493	1.424906	1.753099
		0				1.235277	1.046005	0.850331	0.958385	1.282046	1.584835
		0.05				1.327248	1.129064	0.921836	1.048754	1.408776	1.744046
	1	45°	0.1	0.10	0.1	1.431046	1.222857	1.002577	1.140235	1.539845	1.910240
						1.548917	1.329411	1.094290	1.232662	1.675588	2.084169
						1.683723	1.451312	1.199191	1.325793	1.816371	2.266704
		0				1.327248	1.129064	0.921836	1.048754	1.408776	1.744046
	1	45°	0.1	0.05	0.2	1.331592	1.131558	0.923502	1.050255	1.409461	1.744411
						1.337304	1.134580	0.925440	1.052223	1.410290	1.744834
						1.345131	1.138319	0.927719	1.054911	1.411314	1.745333

List of symbols

α	Aligned angle of magnetic field	ρ_s	Density of nanoparticles
α_{nf}	Thermal diffusivity of magnetic nanofluids	ρ_{nf}	Density of magnetic nanofluids
β_f	Thermal expansion coefficient	$(\rho C_p)_{nf}$	Heat capacity of magnetic nanofluids
γ	Plate inclination angle	$(\rho C_p)_f$	Heat parameters of base fluid
η	Boundary layer thickness	$(\rho C_p)_s$	Heat parameters of nanoparticles
$\theta(\eta)$	Non – dimensional temperature function	$(\rho\beta)_{nf}$	Thermal expansion of magnetic nanofluids coefficient
λ	Velocity ratio parameter	σ	Electrical conductivity
μ_f	Dynamic viscosity of base fluid	τ_w	Wall shear stress
μ_{nf}	Dynamic viscosity of magnetic nanofluids	ϕ	Nanoparticles volume fraction
ρ	Density	$\psi(x, y)$	Stream function
ρ_f	Density of base fluid	$B(x)$	Transverse magnetic field
		B_0	Magnetic field strength

Bi_x	Biot number
$f(\eta)$	Non – dimensional stream function
C_f	Local skin – friction coefficient
Gr_x	Local Grashof number
g	Gravitational acceleration
h_f	Heat transfer coefficient
k_f	Thermal conductivity of base fluid
k_s	Thermal conductivity of nanoparticles.
k_{nf}	Thermal conductivity of magnetic nanofluids
M	Magnetic strength parameter
Nu_x	Local Nusselt number
Pr	Prantl number
q_w	Heat flux
Re_x	Reynolds number
T	Temperature
T_f	Temperature of hot fluid
T_∞	Temperature at the free stream
$U_w(x)$	Plate velocity
U_∞	Velocity in free stream
u	Velocity in x -direction
v	Velocity in y -direction
x	Dimensionless coordinate axis along the inclined plate
y	Dimensionless coordinate axis normal to the surface plate

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Compliance with ethical standards

Conflict of interest

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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