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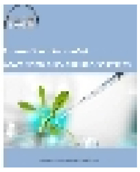

Particle swarm optimization in the coefficients a, b of Forchheimer equation

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ABSTRACT

The Flow is non-laminar in a porous coarse aggregate medium due to velocity increase .Accordingly; the equations based on the layered flows cannot be used as Darcy equation. Thus the equations are required to estimate the hydraulic gradient with respect to the rate of flow and mechanical properties of soil, taking into account the non-linearity of flow. Forchheimer binomial equation is referred to as one of methods which have been accepted very much. In this study, the measured values are calculated using the different methods of Forchheimer equation coefficients with the Experiments on the flow in porous fine rocks in non-lasting conditions for seven soils with different gradation .The method used in this experiment consisted of particle swarm algorithm. One of optimization algorithm is extra-search and it is widely used in engineering affairs. This algorithm is defined based on the movements of birds and fish while hunting. Factors affecting on the estimated coefficients are determined in order to calculate the Forchheimer equation coefficients firstly Using the dimensional analysis and then these coefficients are calculated using the non-dimensional parameters.

1. Introduction

Porous coarse grains mediums are very useful for coastal development, dams' construction, embankment, rip rap and breakwaters. Penetration influx is much more significant than a typical dam in a pebble dam.

Also the leakage forces acting on the aggregate would be quite different with the forces acting on the material on which Darcy flow is flown.

Leakage flow network through porous media varies with flow Reynolds number. Using a non-Darcy reliable equation is important because of non - laminar flow in porous media to determine the leakage forces, flow rate and network flow in the porous structure .Understanding the relationship between velocity and hydraulic gradient is a prerequisite to coarse-grained structural engineering at the crossing flow .The subject of this research is about finding the best equation to describe the flow characteristics in the samples tested in this study as well. Experts of various fields of science and engineering are certainly looking for the better and deeper understanding of the complex behavior of non-laminar flow in porous media during recent decades. The Researchers have proposed various nonlinear models each of which has its own strengths and weaknesses in these years, several investigations including laboratory research, medical research, analytical and numerical calculations. Leakage flow coefficients only depend on the physical parameters of the coarse-grained materials in some provided models such as equations of Wilkins (1956), Martínez (1990), MacCorkindale (1978), Argon (1952), Stephenson (1979) and Martínez (1990). But in other equations such as ward (1964) the coefficients are not determined only by the parameters and it will be necessary to determine the hydraulic conductivity in laboratory. Determination of non-Darcy leakage coefficients are interested and cared in terms of the known parameters of pebble stones media due to the high cost of testing. Various researchers have studied various physical parameters of the porous medium to determine the leak equation coefficients and they have determined their impact on the various experiments with changes on the size, shape and materials (Ergun, 1952).

Since the fluid flow in porous coarse media in common applications is non-Darcy, therefore a linear equation like Laplacian equation cannot be used for engineering analyses. In other words, a linear equation was not established in the material between flow velocity (V) and hydraulic gradient (i) and equation shall be governed as exponential or quadratic equation. Researchers have proposed the various equations to estimate the hydraulic gradient long ago (i) in terms of the mean flow velocity (V) in

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the aggregates that all provided equations can be classified into 2 following general forms:

(1) Binomial equation $i = aV + bV^2$

This equation is known Forchheimer equation and **a** and **b** are the coefficients in it which are a function of the flow characteristics and the porous medium. And they are usually determined by laboratory methods (Wilkins, 1956).

(2) Exponential equation $i = mV^n$

m and **n** are the values dependent on the properties of the porous and fluid medium in the equation (McCorquodale et al., 1978). Simply use of the exponential equation has led to application of various forms of the equation by various researchers [3-5]. Each of these researchers have made the determination of coefficients **m** and **n** subject to a set of characteristics of materials, fluid and flow velocity (or Reynolds number). As a result, there is little consistency between the values of the coefficients provided for a given set of engineering data.

Due to these differences, there is widespread acceptance (regardless of the method used to determine the coefficients for a given material and in the relatively certain range of Reynolds numbers) and we can consider the coefficients constant **m** and **n** with appropriate approximation coefficients (Garga, 1998).

The following individuals each presented a model for nonlinear but lasting analyses in the porous coarse media with personal taste. Laps, 1972, , Senie 1978, Felton and Hara 1995, Furar et al 2004, motsopolos et al. 2005, Cheng et al., 2008.

Khalife and colleagues (2000) studied the impact of porous medium pollution on the permeability coefficient of the material and they provided an equation based on the Hazen formula for it. (Moutsopoulos and Tsihrintzis, 2005). Martinez (2007) conducted a study on the morphology of pores and channels. He realized that the flow characteristics inside the media with spherical particles vary in proportion to the space size and its distribution in the medium. (Cheng et al., 2008).

Rocha and Cruise 2010 began a study in which they had solved the three-dimensional non-compressible fluid leakage inside the porous medium by analytical and literary methods (George and Hansen, 1992).

Belhof et al. 2010 conducted a study about a polynomial equation to describe the flow in porous media in the low Reynolds number range that the proposed equation was as an infinite series of polynomials in terms of flow velocity (Felton and Herrera, 1995).

Zikzing in 2011 investigated the leakage characteristics in the sandstone media with different porosities in laboratory. The result showed that the leakage is closely related to the materials size and rate of pressure on it and structure of pores (Fourar et al., 2004).

Qazi Moradi (2005) conducted a study of the hydraulic gradient in the calculation of aggregate and fragment structures (Cheng et al., 2008).

Azizi et al. (2008) studied the effect of porosity parameters on the hydraulic gradient in Gabion and stepped spillways. They concluded that the porosity effects on the flow pressure drop greater than the downstream slope and energy drop increases with decreasing porosity (Rocha and Cruz, 2010).

Bazargan and Shoaei (2007) conducted a study of non-Darcy flow analysis in the gravel materials using phase variable flow theory. The result is a new equation to obtain the hydraulic gradient used in the phase variable flows theories (Xiexing et al., 2011). Maliknezhad Yazdi and colleagues (2010) conducted a study on the application of adaptive neural fuzzy inference system for hydraulic analysis of flow in the fragment porous media. They came to the conclusion that the model is able to identify the law behind them due to its intelligent structure by calculating the numerical data or examples, without the knowledge of nature and how they function (Balhoff et al., 2010).

Shokri et al. (2011) examined the unsteady flow through a porous pebbles medium experimentally that an equation was proposed in its result (Khalifa et al., 2000).

2. Materials and methods

2.1. Laboratory set in use

The aggregate and gradation curves were prepared to determine the physical parameters. The main components of the device used in this study shown schematically in Figure 1 were: Cistern with a length 13 cm, height 85 cm, width 80 cm, which had a capacity of 10 cubic meters of water. A non-tilt concrete flume was used to the length of 15 m, height 60 cm, width 60 cm, with glass walls and equipped to side overflow with a distance of 4 meters from the top of the flume to provide the constant head on the glass flume and to determine the outflow of the glass flume from triangular overflow with an angle of 90 degrees to do the experiments which the reason of using this type of overflow is a measurable height at low flows. A Pump with a flow rate of 25 liters per second is used to create the flow in the flume. Flow control is done by a micro molinete. The board of this laboratory flume Piezometers consists of 13 piezometers. The distance between piezometers is equal to 20 cm. Tools of Grading materials include the balance and Shaker shown in Fig.4.

3. Forchheimer model

Two general methods are used to estimate the hydraulic gradient in terms of velocity, Forchheimer and exponential equation. The equation Forchheimer was used in this study. Forchheimer expressed for the first time the equation between the hydraulic gradient and mean flow velocity in 1901 with the following equation.

$$i = aV + bV^2 \quad (3)$$

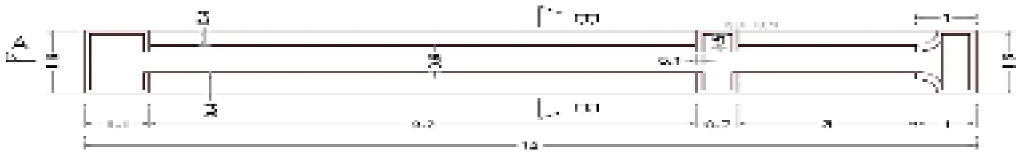

Fig. 1: Wave flume plane used in this study (values in meters)

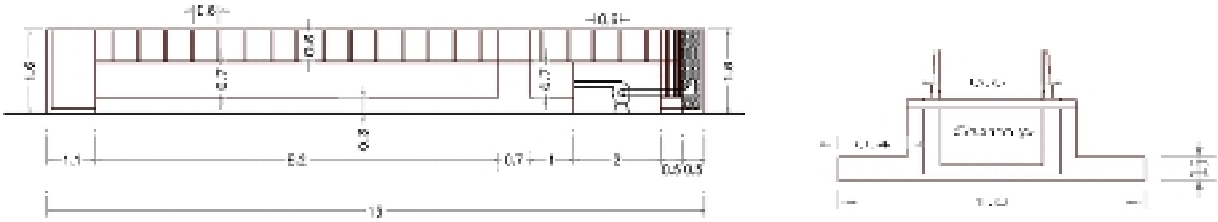

Fig. 2: Section B-B (transverse) of flume and its cistern

Fig. 3: Section A-A (Longitudinal) flume and its cistern

Fig.4: balance and Alignment of sieves and shake for shaking, Figure 5: Viewing the freatik line of flow crossing through the material

In the above equation, i: hydraulic gradient, V: velocity and **a** and **b** of equation Forchheimer coefficients. Later it was confirmed by several researchers from the viewpoint of theoretical validity. They attempted to link the parameters **a** and **b** to the physical properties of the fluid and porous media in their research and their work outcome is presented in the form of multiple equations (Martins et al., 2007).

4. Dimensional analysis

The purpose of this study in equation Forchheimer is to present the coefficients **a** and **b**. Forchheimer considered these two parameters as constant numbers in their equations , Since other parameters involved in determining the hydraulic gradient other than velocity . Effective Parameters are considered in determining two parameters **a** and **b** in this study and effective parameters are assumed using dimensional analysis using dimensional analysis. And then some parameters are used directly after performing the sensitivity analysis on the equation. And some parameters also show their effect on other parameters indirectly. Important parameters in determining the hydraulic gradient can be represented as the following equation:

$$i = f(R, V, \mu, g, k, \rho, D, f, n, D_{10}, D_{30}, D_{60}, C_u, C_c) \quad (4)$$

In the above equation: I- hydraulic gradient. R - Hydraulic radius. V- Velocity of flow through the channel. μ- Fluid dynamic viscosity. G- Acceleration of gravity. K- intrinsic permeability of materials. ρ - fluid density. D -hydraulic diameter. F- friction coefficient. N- Porosity. D10, D30 and D60, respectively diameters of 10, 30 and 60 percent of their smaller units. Cu form factor. Cc Curvature coefficient. Dimensionless parameters from the above equation can be represented as the following equation:

$$i = f(\frac{\rho VD}{\nu} C_u, C_c, n) \quad (5)$$

Since the Reynolds number $\frac{\rho VD}{\nu}$ is not of considerable importance in open channel flow. Therefore, this parameter is not intended to estimate the hydraulic gradient. In this study, equation below is used to estimate the coefficients that can be expressed in three different structures. And the constant coefficients are estimated using particle swarm optimization algorithm.

$$a \ \& \ b = f(n, C_u, C_c) \quad (6)$$

The objective function definition is the first step in numerical analysis that we measure the level of competence of the proposed formula by it. The purpose used in this research is function squares root mean error which is a criterion accepted and widely used in adaptive computing and data processing.

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}\left(X^{i}_{lab} - X^{i}_{formula}\right)^2}{n}} \quad (7)$$

xlab and xformula in the above formula are laboratory data and the result obtained from the proposed structure respectively . N are the total number and the data value will be equal to seven in the research given the 7 laboratory data. The equation calculates the errors between the laboratory value and the resulting formula for all seven materials and it collects them together and reports the results in the general case. We expect (as the objective function) the reduced amount of error for the proposed structures. Then, different structures are available to provide different structures due to the nonlinearity of the equation used in this study to determine the coefficients **a** and **b**. There are several structures as first and second structure to provide these structures by considering the effective parameters and using dimensional analysis. And other structure is provided as polynomial whose structure is also proposed based on algorithm in this study (Particle swarm).

Analysis and discussion

5. First structure

This is the first and simplest structure in terms of appearance. The following equations are Proposed for the coefficients Forchheimer

$$a = X_1 n + X_2 D_{10} + X_3 D_{30} + X_4 D_{60} + X_5 C_u + X_6 C_c \quad b = Y_1 n + Y_2 D_{10} + Y_3 D_{30} + Y_4 D_{60} + Y_5 C_u + Y_6 C_c$$

Parameters X1, X2 ... Y6 are the decision variables of optimization process or the same unknowns to obtain the best values in the above equations. n (porosity) is a linear combination of independent parameters. Cu Form factor and Cc curvature coefficient and D10, D30 and D60 (diameters that ten percent, thirty percent, and sixty percent of materials are its smaller units). Particle swarm algorithm is used in this step. The unknown coefficients objective function equation is obtained according to the program written in BASIC. Table 1 shows the results of an evolutionary algorithm proposed in this study (Particle Swarm) for the coefficients a and b as given equations. All other parameters are zero in the A parameter shown except coefficients X1 and X 5. Coefficients Y2 and Y6 have values in the parameter b. The following equations can be written as the following with general form of equation Forchheimer for calculation of hydraulic gradient according to the values presented in the above table for the particle swarm algorithm.

5.1. Particle swarm algorithm:

$$a = 79.18\,n + 2.295\,C_u \quad (9)$$

$$b = 0.076\,D_{10} + 0.361\,C_c \quad (10)$$

$$i = (79.18n + 2.295C_u)V + (0.076D_{10} + 0.361C_c)V^2 \quad (11)$$

Based on the following results table, the values of some X and Y in these structures have become zero using the particle swarm algorithm because of lack of impact of their coefficients.

Table 1: Optimized values of unknown parameters of first proposed structure equations

		Particle swarm optimization (PSO)			Particle swarm optimization (PSO)
Coefficient a	X1	79.117	Coefficient b	Y1	0
	X2	0		Y2	0.0760
	X3	0		Y3	0
	X4	0		Y4	0
	X5	2.295		Y5	0
	X6	0		Y6	0.3609
	RMSE error value	1210.694		RMSE error value	1.808

The proposed equations of First structure can be used to calculate the Forchheimer coefficients and comparing the results with experimental data obtaining the unknown values by particle swarm method.

Table 2 the coefficients obtained by using algorithm shows the particle swarm for every seven soils used in this study. Thus the modified Forchheimer equation can be provided for calculating the hydraulic gradient by means of equation Forchheimer as i = av + bV2 and use of below table for each of the proposed gradations.

Fig. 1 comparison of experimental data and the results from the first coefficient estimate of equation Forchheimer (a) is shown for different soils in the first structure using evolutionary algorithms.

According to the figure, it is observed that the results are in good agreement with the experimental presented values using particle swarm algorithm except the soil type 1 and type 7.

Table 2: Calculation of Forchheimer coefficients values by optimized parameters

	Soil 1	Soil 2	Soil 3	Soil 4	Soil 5	Soil 6	Soil 7
Laboratory values of coefficient a	79.89	25.34	31.78	24.34	34.06	39.79	76.21
Computational value of a (Particle swarm optimization)	41.21	31.98	36.86	32.51	36.70	36.67	36.80
Laboratory values of coefficient b	0.9946	1.347	0.82	3.159	0.8758	0.504	0.8631
Computational value of b (Particle swarm optimization)	0.7823	1.3348	0.8130	1.0179	1.1122	0.7848	1.0030

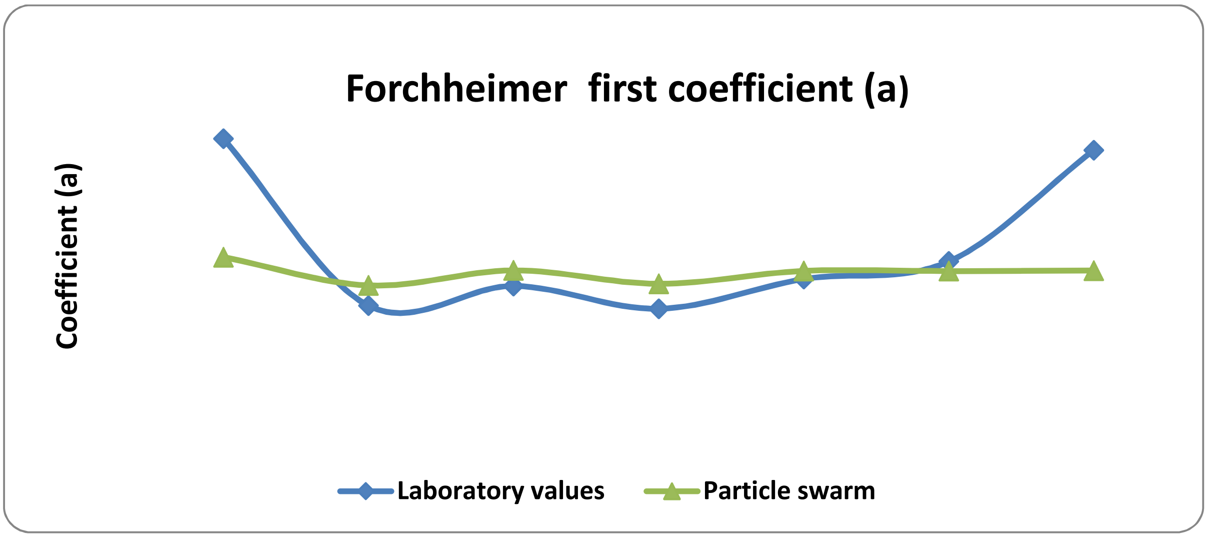

Fig. 4: Comparison of the results of the first structure formulas for first Forchheimer coefficients

Fig.2 particle Swarm is shown by comparing the experimental data and the results from the estimation of the second coefficient of equation Forchheimer (b) for the first structure using the algorithm method. According to this figure, it can be seen that the estimates are done accurately. As roughly estimated values and the values of the experimental results are nearly equal except the amount determined for the soil 4.

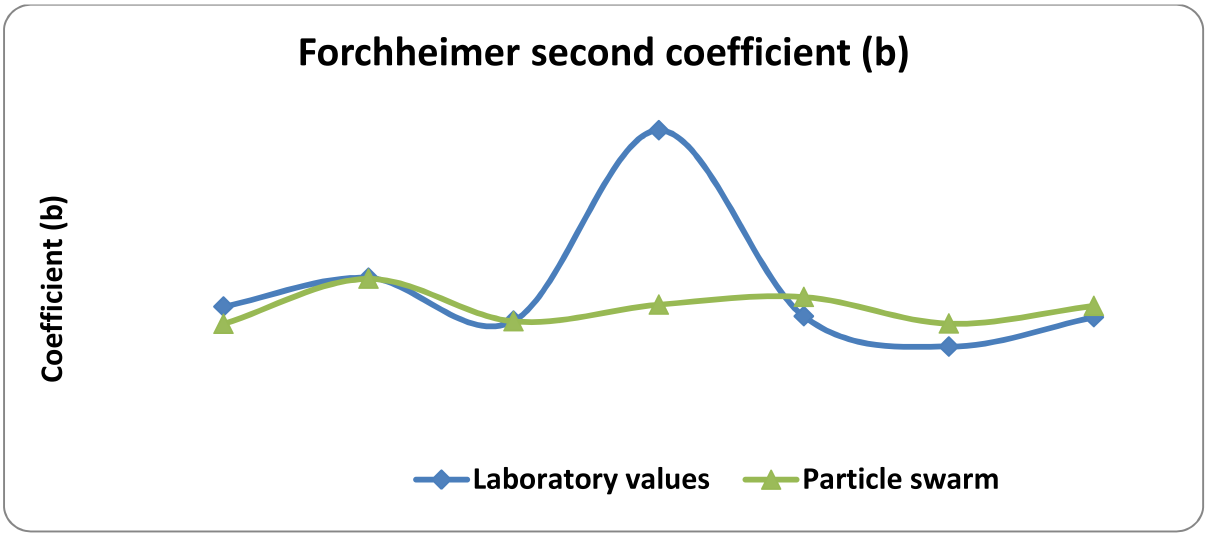

Fig. 5: Comparison of the results of first structure formulas for the second Forchheimer coefficients

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6. Second structure

We test the exponent 2 for the input characteristics in the second structure that the shape of this structure is similar to the first structure. Only difference of the second order (or structure) is the effective parameters in the estimation of coefficients a and b are defined as following.

The results from evolutionary algorithm offer the particle swarm. The purpose of this algorithm is to calculate the coefficients a and b are given in the following equations.

$$a = X_1 n^2 + X_2 D_{10}^{\ 2} + X_3 D_{30}^{\ 2} + X_4 D_{60}^{\ 2} + X_5 C_u^{\ 2} + X_6 C_c^{\ 2} \quad (12)$$

$$b = Y_1 n^2 + Y_2 D_{10}^{\ 2} + Y_3 D_{30}^{\ 2} + Y_4 D_{60}^{\ 2} + Y_5 C_u^{\ 2} + Y_6 C_c^{\ 2}$$

According to the table it can be seen that the parameter of coefficients X1 and X5 (coefficients of parameters n and Cu) and the parameter b of coefficients Y2, Y5 and Y6 have a value. So according to the values presented in this table and equations, we can write the following equations for the particle swarm algorithm in the following figure keeping the general form of equation Forchheimer to calculate the hydraulic gradient:

Particle swarm algorithm:

$$a = 186.02n + 0.407C_u \quad (13)$$

$$b = 0.007D_{10} + 0.82C_u + 0.14C_c \quad (14)$$

$$i = \left(186.02\,n + 0.407\,C_u\right)\,V + \left(0.007\,D_{10} + 0.82\,C_u + \right. \quad (15)$$

The results presented in the following tables are shown using the particle swarm algorithm. As per both equations, the coefficients a and b in the optimization algorithm are related to the same parameters. And the coefficients Xi and Yi provided for each of them are also relatively equal.

Table 3: Optimized values of the unknown parameters of 2nd proposed structural equations

		Particle swarm optimization (PSO)			Particle swarm optimization (PSO)
Coefficient a	X1	186.019	Coefficient b	Y1	0
	X2	0		Y2	0.0070
	X3	0		Y3	0
	X4	0		Y4	0
	X5	0.407		Y5	0.081655
	X6	0		Y6	0.1401
	RMSE error value	2659.269		RMSE error value	3.904

One of the main objectives of the research is to provide the coefficients of equation Forchheimer a and b (table 4) for different soils given the mechanical characteristics and soil gradation. Therefore the following equations are used to calculate the hydraulic gradient using the equation Forchheimer and following table for each provided gradations.

Table 4: Calculating the values of Forchheimer coefficients by optimized parameters

	Soil 1	Soil 2	Soil 3	Soil 4	Soil 5	Soil 6	Soil 7
Laboratory values of coefficient a	79.89	25.34	31.78	24.34	34.06	39.79	76.21
Computational value of a (Particle swarm optimization)	42.72	25.43	31.95	26.37	34.70	32.86	32.74
Laboratory values of coefficient b	0.9946	1.347	0.82	3.159	0.8758	0.504	0.8631
Computational value of b (Particle swarm optimization)	0.3099	0.3383	0.3475	0.3272	0.3142	0.3099	0.3664

Figure 8 comparisons of experimental values and the results from first coefficient of equation Forchheimer (a) is calculated in the second structure using particle swarm algorithm. The Results of estimates conducted by this algorithm is similar to the first given the figure. This figure does not enjoy the relatively good estimates for the soils 1 and 7. While there is a good agreement for other soils roughly in the estimated values with actual values.

Fig. 7 comparison of experimental data and the results from estimate of the second coefficient of equation Forchheimer (b) in the second structure shows the particle swarm for seven soils with different gradation using the algorithm. According to the figure, the estimated results of this coefficient by used algorithm are the same with the laboratory results almost for all soils except the soil 4.

7. Third structure

Only parameters n (porosity) and Cu (form factor) effect on the equation and other parameters have zero coefficient to define the third structure According to the results obtained from both previous structures. It can be seen in the equation of Forchheimer first coefficient i.e. coefficient a in both previous structures. Thus, these two quantities are used for coefficient a in the third proposed structure.

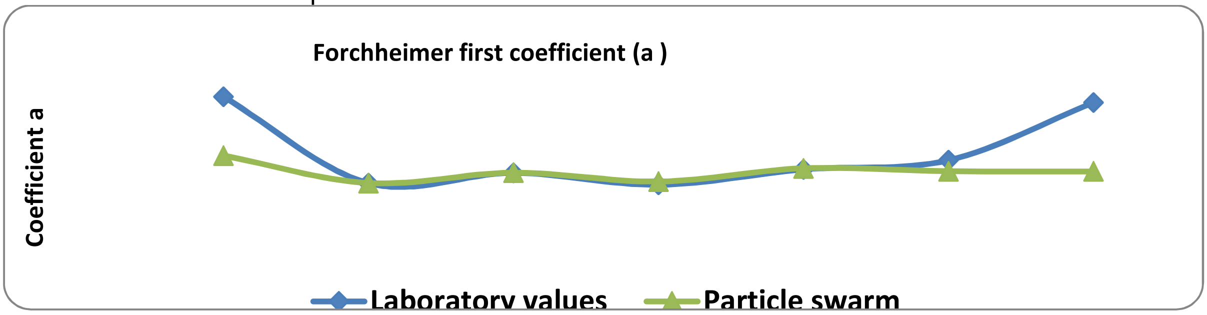

Fig. 6: Comparison of the results of the secondary structure formula for Forchheimer first coefficient

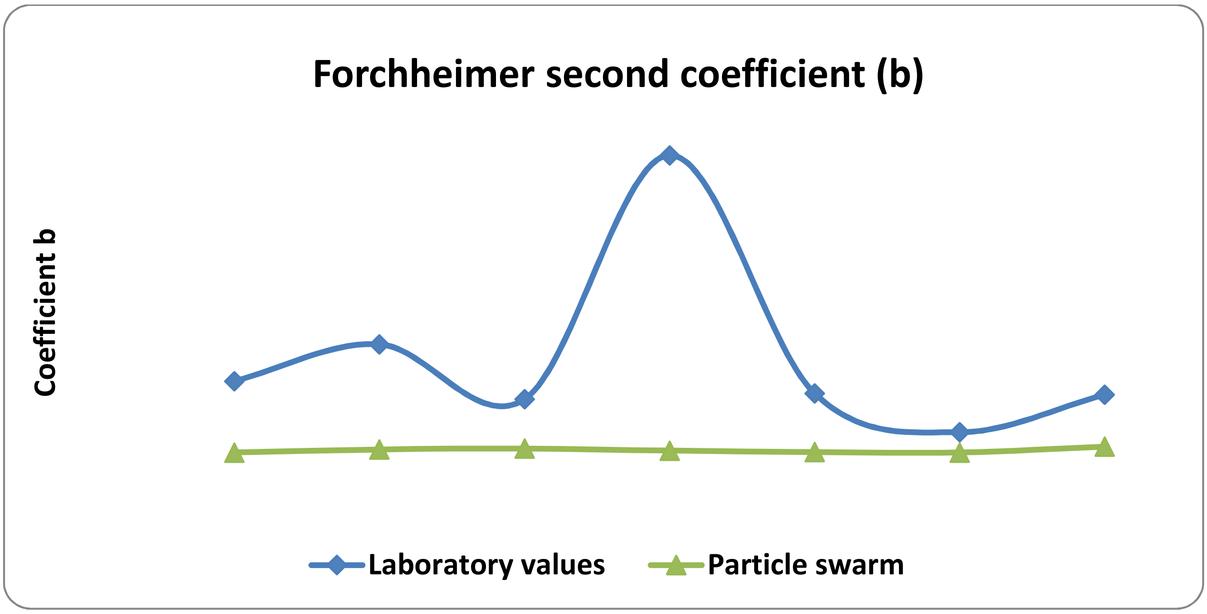

Fig. 7: Comparison of the results of second structure formula for Forchheimer second coefficient

And in this phase, their coefficient and exponent can be considered unknown. In addition, the product of these quantities is included in the new structure to get the better fit in the evolutionary algorithm. Equation b is defined by the same reasoning:

$$a = X_1 n^{X_2} + X_3 C_u^{X_4} + X_5 n C_u \quad (16)$$

$$b = Y_1 D_{10}^{Y_2} + Y_3 C_c^{Y_4} \quad (17)$$

The results of the optimization process on the mentioned equations are as following:

Table 5: Optimized values of the unknown parameters of third proposed structure

		Particle swarm optimization (PSO)			Particle swarm optimization (PSO)
Coefficient a	X1	10228549	Coefficient b	Y1	0.076
	X2	16.56019		Y2	0.9880
	X3	1.258109		Y3	0.3599
	X4	0.5019981		Y4	0.9807
	X5	46.890		Y5	
	X6			Y6	
	RMSE error value	523.138		RMSE error value	1.831

Table 6: Calculation of values of Forchheimer coefficients by optimized parameters

	Soil 1	Soil 2	Soil 3	Soil 4	Soil 5	Soil 6	Soil 7
Laboratory values of coefficient a	79.89	25.34	31.78	24.34	34.06	39.79	76.21
Computational value of a (Particle swarm optimization)	80.41	25.67	46.72	26.15	34.29	40.24	42.20
Laboratory values of coefficient b	0.9946	1.347	0.82	3.159	0.8758	0.504	0.8631
Computational value of b (Particle swarm optimization)	0.7727	1.3044	0.8018	1.0010	1.0912	0.7747	0.9865

Fig.10 Particle swarm is shown for seven different soils by comparing the experimental results with those values obtained from the first coefficient estimates of equation Forchheimer using evolutionary algorithms. Accuracy of estimates of the third structure is more than the first and second structures depending on the figure. And relatively good results are provided for most soils and Laboratory values and estimated values are in good agreement in all soils except 3 and 7.

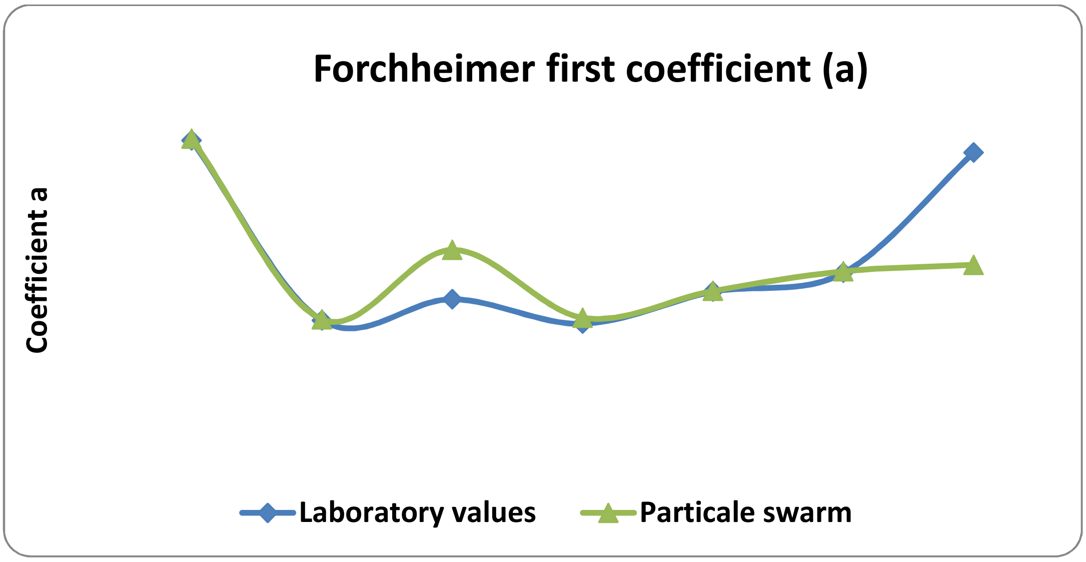

Fig. 8: Comparison of the results of the tertiary structure formulas for Forchheimer first coefficients

Fig.11 the particle swarm is shown by comparing the experimental results with the values obtained from second coefficient estimate of equation Forchheimer using the third structure provided in this study and evolutionary algorithm. The results are the same nearly for other soils except the soil 4 according to the Fig.4.

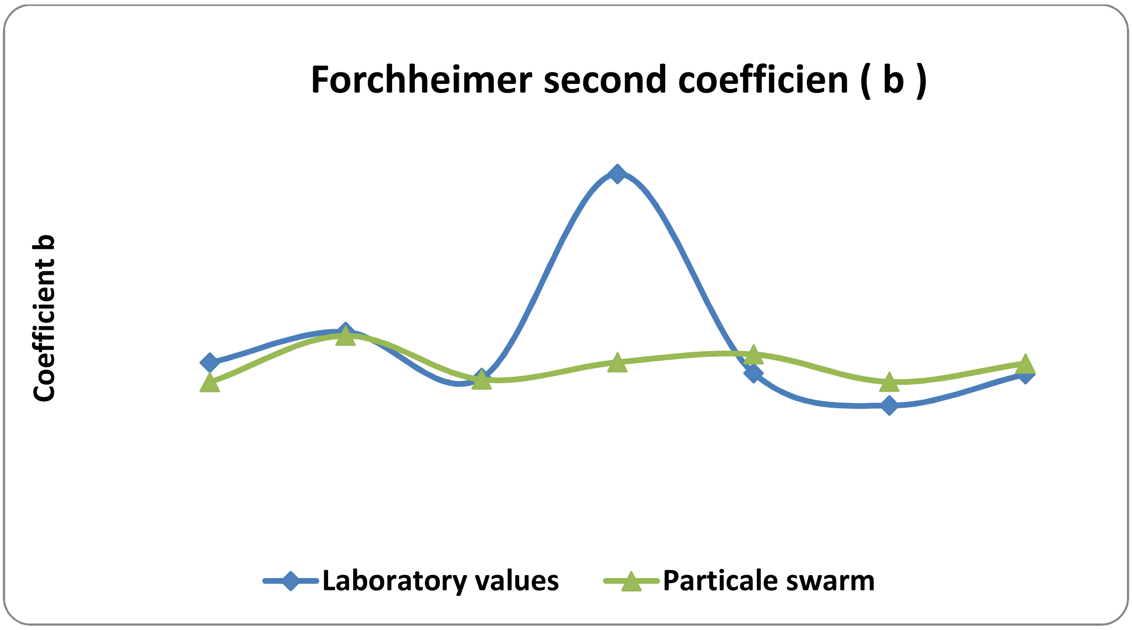

Fig. 9: Comparison of the results of tertiary structure formulas for Forchheimer second coefficients

8. Conclusion

The best equations for Forchheimer coefficients can be chosen exploring the three structures described in this study that the first coefficient of equation Forchheimer (a) is worth to mention using the third structure and the second coefficient of this equation (b) using the first structure. These equations can predict the values of the first and second coefficients of the equation Forchheimer in terms of materials mechanical properties. These equations are provided as follows:

(18)
$$a = 10220420\ n^{16.56} + 1.259\ C_u^{0.501} + 46.408\ nC_u$$

(19) $$b = 0.0767\ D_{10} + 0.3643\, C_c$$

$$i = (10220420\, n^{16.56} + 1.259\, C_u^{0.501} + 46.408\, nC_u)V + (0.0767 D_{10} + 0.3643 C_c)\ V^2 \quad (20)$$

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